A Simplified Extension of X-parameters to Describe Memory Effects for Wideband Modulated Signals

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Abstract—An original way is presented to model memory effects of microwave amplifiers in the case of wideband modulated signals. The model is derived as a limiting case of the more general dynamic X-parameter theory. For a given component, the model is identified from pulsed envelope X-parameter measurements performed with an NVNA. The resulting nonlinear X-parameter model is quantitatively described by a 2-variate kernel function that enables the derivation of an optimal static AM-AM AM-PM characteristic for every possible input envelope probability density function. The model is validated by performing a set of 2-tone experiments. The model can be implemented in the ADS circuit envelope simulator.

Index Terms—behavioral model, memory effects, frequency domain, pulsed envelope, measurements, X-parameters, NVNA

I. INTRODUCTION

Behavioral modeling of microwave components is of great interest to the designers of amplifiers that are used in today’s wireless communication infrastructure. An important problem faced by these engineers is the difficulty to characterize, describe mathematically, and simulate the nonlinear behavior of amplifiers that are stimulated by signals that have a high peak-to-average ratio and that excite the amplifier over the full operating range of instantaneous power. This is problematic for at least two reasons. Firstly, the amplifier behavior may be driven into full saturation and is as such strongly nonlinear. Secondly, the amplifier behavior shows long-term memory effects. Such memory effects are caused by a variety of time-varying operating conditions, such as dynamic self-heating and bias-line modulation. These changes are induced by the input signal itself and vary at a relatively slow rate compared to the modulation speed. As a consequence the instantaneous behavior of the amplifier becomes a function not only of the instantaneous value of the input signal, but also of the past values of the input signal. This is referred to as a "long term memory effect".

One of the consequences of long term memory effects is that a simple AM-AM AM-PM characteristic, typically measured under steady state CW operating conditions at the carrier frequency, is not accurate enough to describe the amplifier behavior under actual operating conditions. In a more advanced existing approach [1], one excites the amplifier with a realistically modulated signal and one fits an optimal AM-AM AM-PM characteristic to the set of measured samples of the input and the corresponding output envelope. Although the resulting behavioral model is often much more accurate in describing the amplifier behavior than a model derived from steady state CW measurements, the disadvantage of such a modeling approach is that the model is only valid if the probability density function of the applied signal is almost identical to the probability density function of the test signal that was used for fitting the AM-AM AM-PM characteristic. This limits the practical use of such models in a design environment where one may not precisely know the probability density function of the applied signals beforehand. In this paper we will present a new modeling approach, quantitatively described by a 2-variate kernel function that enables the accurate prediction of an optimal AM-AM AM-PM characteristic for any possible input amplitude probability density function. As will be shown in the following, the 2-variate kernel function can be measured by performing pulsed envelope X-parameter measurements on a modern Nonlinear Vector Network Analyzer (NVNA) [5].

The new modeling approach is derived as a simplification of the so called “dynamic X-parameter” model, as described in [2], which was based on the earlier Poly-Harmonic Distortion (PHD) and X-parameter work described in [3] and [4]. The simplified version of the abovementioned dynamic X-parameter model will be valid for wideband modulated signals or, equivalently, fast varying input envelope signals. Note that “fast” is to be considered relative to the time constants of the nonlinear dynamic baseband effects causing the long term memory effects like e.g. thermal, biasing or trapping effects.

II. MODEL THEORY

As described in [2] memory effects can be introduced by making use of one or more hidden variables. The idea is that, in a system with memory, the mapping from the input signal to
the output signal is no longer a function of the input signal amplitude only, but is also a function of an arbitrary number \( N \) of \textit{a priori} unknown hidden variables, denoted \( h_1(t), h_2(t), \ldots, h_N(t) \). These variables represent time varying physical quantities inside the component, for example, temperature, bias voltages or currents, or semiconductor trapping phenomena that influence the mapping from the input RF signal to the output signal. For simplicity we deal with a unilateral and perfectly matched device and neglect all harmonics. Extensions to multi-port devices with mismatch and harmonics will be treated elsewhere.

The time-dependent envelope of the scattered wave \( B(t) \) is given by a generic nonlinear X-parameter function \( X_{21}(.) \) of the input amplitude envelope \( A(t) \) and the time-dependent values of all relevant hidden state variables, \( h_i(t) \), as described by

\[
B(t) = X_{21}\left([A(t)], h_1(t), h_2(t), \ldots, h_N(t)\right) \Phi(t)
\]

with \( \Phi(t) = e^{i \phi(t)} \).

As described in [2] the dependence of \( B(t) \) on the phase of \( A(t) \) as modeled by (1) is a good approximation for many systems and its limitation will not be considered further here.

The black-box assumption about the relationship between the input signal \( A(t) \) and the hidden variables \( h_i(t) \) is mathematically expressed as

\[
h_i(t) = \int_0^\infty P_i\left(|A(t-u)|\right) k_i(u) \, du.
\]

Equation (2) expresses that the \( i \)-th hidden variable is generated by a linear filter operation, characterized by its impulse response \( k_i(.) \), that operates on a nonlinear function \( P_i(.) \) of the input signal amplitude \( |A(t)| \). \( P_i(.) \) can be interpreted as a source term that describes how the input signal is related to the excitation of a particular hidden variable. For example, \( P_i(.) \) could describe the power dissipation as a function of the input signal amplitude, whereby \( h_i(.) \) is the temperature. The impulse response \( k_i(.) \) describes the actual dynamics of a hidden variable, e.g. the thermal relaxation.

Consider now the simplified case where the input envelope is varying fast compared to the time constants associated with the impulse responses \( k_i(.) \). As shown in the Appendix, if the statistical properties of the input envelope are stationary the convolution described by (2) can be approximated as follows:

\[
h_i(t) \approx K_i(\infty) E\left[P_i(a)\right],
\]

with “a” representing the random variable corresponding to the input amplitude \( |A(t)| \), with \( E(.) \) representing the statistical expectation operator and with \( K_i(t) \) being equal to the step response corresponding to the impulse response \( k_i(t) \). Equation (3) shows that the hidden variables are no longer varying over time but have a fixed “frozen” value. Substitution of (3) in (1) results in

\[
B(t) \approx X_{21}\left([A(t)], K_1(\infty) E(P_1(a)), K_2(\infty) E(P_2(a)), \ldots\right) \Phi(t).
\]

As mentioned in [2] the X-parameter function \( X_{21}(.) \) can in many practical cases be approximated by a function which is linear in the hidden variables. This can be expressed as

\[
X_{21}\left([A(t)], K_1(\infty) E(P_1(a)), K_2(\infty) E(P_2(a)), \ldots\right) \approx
\]

\[
X_{21}\left([A(t)], 0, \ldots\right) + \sum_{i=1}^N D_i\left([A(t)]\right) K_i(\infty) E(P_i(a)),
\]

with

\[
D_i(x) = \frac{\partial X_{21}}{\partial h_i}(x,0,0,\ldots).
\]

As the expectation operator is a linear operator, regardless of the nature of the random variables it is operating on, one can write

\[
X_{21}\left([A(t)], 0, \ldots\right) + \sum_{i=1}^N D_i\left([A(t)]\right) K_i(\infty) E(P_i(a)) =
\]

\[
E\left[X_{21}\left([A(t)], 0, \ldots\right) + \sum_{i=1}^N D_i\left([A(t)]\right) K_i(\infty) P_i(a)\right].
\]

Substitution of (7) into (5) results in

\[
X_{21}\left([A(t)], K_1(\infty) E(P_1(a)), K_2(\infty) E(P_2(a)), \ldots\right) \approx
\]

\[
E\left[X_{21}\left([A(t)], K_1(\infty) P_1(a), K_2(\infty) P_2(a), \ldots\right)\right].
\]

We define a 2-variate kernel function \( X_{ST}(.) \) given by

\[
X_{ST}(x,y) = X_{21}\left(x, K_1(\infty) P_1(y), K_2(\infty) P_2(y), \ldots\right).
\]

Substitution of (9) into (8) and (8) into (4) results in

\[
B(t) \approx E\left(X_{ST}\left([A(t)], a\right)\right) \Phi(t).
\]

Note that the expectation operator \( E(.) \) in (10) can also be written as an explicit functional of the probability density function of the random variable “a”, noted as \( p(a) \), which leads to

\[
B(t) \approx \int_{0}^{\infty} X_{ST}\left([A(t)], a\right) p(a) \, da \Phi(t).
\]

Expressed in words: the resulting behavioral model described by (11) is a static AM-AM AM-PM transfer characteristic that is generated by taking a weighted average of a 2-variate \( X_{ST}(.) \) kernel function, using the probability density function of the input amplitudes \( p(.) \) as a weighting function.

It is important to note that once the 2-variate kernel function is known the model described by (11) enables the calculation of a static transfer characteristic, or in other words, a static
mapping from $A(t)$ to $B(t)$, for any arbitrary input amplitude distribution.

### III. MODEL IDENTIFICATION

#### A. Theory

The goal of the nonlinear model identification is to determine this $X_{ST}(\cdot)$ kernel function by a series of experiments. One way to do this is explained and demonstrated in the following.

Consider that one wants to find the value of $X_{ST}(A_1, A_2)$. In order to find this value one applies constant envelope amplitude of $A_2$ up to time zero and one switches the value of the input envelope amplitude, at time zero, to a value $A_1$. The value of the output $B(t)$ right after the switching has taken place (at time “$\varepsilon$”) will be equal to the value $X_{ST}(A_1, A_2)$. Note that one will assume in the following that the applied signals have zero phase such that $\Phi = 1$.

Indeed, using (2), one finds that, for constant input amplitude of $A_2$ up to time zero

$$h_i(\varepsilon) = \int_{0}^{\infty} P_i(A_2) k_i(u) du = K_i(\infty) P_i(A_2),$$

such that, using (1)

$$B(\varepsilon) = X_{ST}(A_1, K_i(\infty) P_i(A_2), \ldots)$$

and finally, using (9)

$$X_{ST}(A_1, A_2) = B(\varepsilon).$$

Repeating this pulsed RF measurement for a range of $A_1$ and $A_2$ that covers the potential operating range of the device-under-test (DUT) results in the complete knowledge of the 2 dimensional model kernel $X_{ST}(\cdot)$.

#### B. Experiment

The performance of the model is tested on a ZHL-11AD+, a packaged microwave amplifier from Mini-Circuits. This device-under-test (DUT) has a specified bandwidth from 2MHz to 2GHz, a gain of 11dB and a saturated output power level of 3dBm. The carrier frequency chosen for the experiment is 1750MHz. An Agilent NVNA with pulsed envelope option is used for the pulsed measurements needed for model extraction as well as for the 2-tone model verification measurements.

First, the DUT is characterized by applying an extended set of large signal input steps and by measuring the response of the DUT as soon as possible after applying the pulse. Note that each of these large signal steps differs from a classic pulse measurement in the sense that the initial signal amplitude, before the pulse, is different from zero. This initial amplitude level corresponds to the variable $A_2$ in (14). The amplitude after the step corresponds to the variable $A_1$. For our measurements “$\varepsilon$” is equal to 50ns as the B-wave is measured 50ns after the application of the step. Sweeping both the $A_1$ and the $A_2$ level results in a direct measurement of the 2-dimensional kernel function $X_{ST}(\cdot, \cdot)$. For our specific measurement the peak amplitudes of $A_1$ and $A_2$ have been swept up to a level of 0.205V (-3.8dBm) in 18 uniform steps. The amplitude of the measured kernel function $X_{ST}(A_1, A_2)$ for the full sweep for $A_2$ and for a set of 6 fixed values $A_1$ is illustrated in figure 1.

![Figure 1. Amplitude of kernel function $X_{ST}(\cdot, \cdot)$](image1.png)

The phase of the measured kernel function $X_{ST}(A_1, A_2)$ for the full sweep for $A_2$ and for a fixed value of $A_1$ equal to -3.8dBm is illustrated in figure 2.

![Figure 2. Phase of kernel function $X_{ST}(\cdot, \cdot)$](image2.png)

Figure 1 and figure 2 show that the amplitude and the phase of the output does not only depend on the instantaneous input amplitude $A_1$ but also on the value of the initial input amplitude $A_2$ (the input amplitude prior to the application of $A_1$). As can be seen in figure 1, a high initial value of $A_2$ causes a significant compression of the gain, even for low level instantaneous inputs. In other words, the overall gain is significantly lower right after the amplifier has been saturated.

### IV. MODEL VERIFICATION

Once the 2-dimensional kernel $X_{ST}(\cdot)$ has been derived from pulsed RF measurements, the actual behavioral model, as given by (10), is verified by performing a set of 2-tone
verification measurements. These measurements are also performed on the Agilent NVNA. A tone spacing, denoted \( f_{\text{MOD}} \), is chosen of 3.75MHz, symmetric with respect to the RF carrier at 1750MHz. The 2-tone measurements are performed at 4 power levels, namely -20dBm, -15dBm, -12dBm and -10dBm per tone. The 2-tone input signal is generated by combining the 2 synthesizers of the NVNA. The instrument applies the input signal and actually measures the amplitude and the phase of all the relevant tones present in the input as well as in the output signal. The spectrum of the measured input signal (the A-wave) and the corresponding output signal (the B-wave) for the highest input power level are shown in figures 3 and 4. Significant intermodulation products are present in the output signal.

As the model expressed by (10) operates in the envelope domain, any 2-tone input signal is first to be expressed as a complex envelope. For a peak amplitude of one of the tones equal to "A", the input envelope representation \( A(t) \) is given by

\[
A(t) = 2A \cos \left( \pi f_{\text{MOD}} t \right).
\]  

Equation (10) is then used to calculate the output envelopes that correspond to the given input envelope \( A(t) \). For a practical implementation the expectation operator of (10) is approximated by a weighted average over one period of a uniformly sampled version of the periodic input envelope. For N samples, and with \( A[i] \) representing the \( i^{th} \) sample, one can calculate a sampled version of the modeled output envelope \( B_{\text{MOD}}[i] \) as follows:

\[
B_{\text{MOD}}[i] = \left( \frac{1}{N} \sum_{j=1}^{N} X_{\text{ST}} \left( |A[i]|, |A[j]| \right) \right) e^{j\omega t[i]}
\]  

(16)

The uniformly sampled modeled complex envelope \( B_{\text{MOD}}[i] \) can then be compared with the uniformly sampled measured output envelope \( B_{\text{MEAS}}[i] \). To highlight the performance of the new model, the result of using a traditional static AM-AM AM-PM characteristic (represented by the traditional static \( X_{21} \) parameter) is also calculated. The corresponding output is called \( B_{\text{SS}}[i] \), it is simply calculated as

\[
B_{\text{SS}}[i] = X_{21} \left( |A[i]| \right) e^{j\omega t[i]}
\]  

(17)

The resulting output envelopes for the highest input power are shown in figure 6. Figures 6 clearly shows that the complex envelopes as calculated by using the new model are much more accurate than the envelopes that are calculated by using the static \( X_{21} \) parameter (which is equivalent to a traditional AM-AM AM-PM characteristic).
Using a discrete Fourier transform on the modeled output envelopes results in the modeled spectra. This allows us to verify the capability of the model to predict 2-tone compression characteristics and intermodulation products. The modeled 2-tone compression characteristic and IM3-products are shown in figures 8 and 9. These figures clearly show that the new model can predict the 2-tone compression characteristic and the IM3 products with great accuracy and as such outperforms a traditional static AM-AM AM-PM characteristic.

Tables 1 to 3 contain the measured and modeled 2-tone gain, IM3 products and IM5 products for the 4 input power levels.

Table 1. Comparison of measured and modeled gain

<table>
<thead>
<tr>
<th>Input power (dBm per tone)</th>
<th>-20</th>
<th>-15</th>
<th>-12</th>
<th>-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured gain (dB)</td>
<td>10.41</td>
<td>9.84</td>
<td>8.85</td>
<td>7.97</td>
</tr>
<tr>
<td>Stochastic model gain (dB)</td>
<td>10.39</td>
<td>9.71</td>
<td>8.60</td>
<td>7.65</td>
</tr>
<tr>
<td>Static model gain (dB)</td>
<td>10.35</td>
<td>9.52</td>
<td>8.19</td>
<td>7.07</td>
</tr>
</tbody>
</table>

Table 2. Comparison of measured and modeled IM3

<table>
<thead>
<tr>
<th>Input pow. (dBm per tone)</th>
<th>-20</th>
<th>-15</th>
<th>-12</th>
<th>-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured IM3 (dBm)</td>
<td>-53.0</td>
<td>-35.5</td>
<td>-28.9</td>
<td>-28.0</td>
</tr>
<tr>
<td>Stoch. model IM3 (dBm)</td>
<td>-52.4</td>
<td>-35.1</td>
<td>-28.5</td>
<td>-27.2</td>
</tr>
<tr>
<td>Static model IM3 (dBm)</td>
<td>-48.0</td>
<td>-31.0</td>
<td>-23.1</td>
<td>-20.1</td>
</tr>
</tbody>
</table>

Table 3. Comparison of measured and modeled IM5

<table>
<thead>
<tr>
<th>Input pow. (dBm per tone)</th>
<th>-20</th>
<th>-15</th>
<th>-12</th>
<th>-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured IM5 (dBm)</td>
<td>-82.0</td>
<td>-68.4</td>
<td>-42.8</td>
<td>-36.1</td>
</tr>
<tr>
<td>Stoch. model IM5 (dBm)</td>
<td>-71.2</td>
<td>-67.5</td>
<td>-42.8</td>
<td>-36.9</td>
</tr>
<tr>
<td>Static model IM5 (dBm)</td>
<td>-67.4</td>
<td>-53.8</td>
<td>-44.0</td>
<td>-33.5</td>
</tr>
</tbody>
</table>

Except for the lowest power level IM5 product, which is near the noise floor of our measurements, the new stochastic model predicts gain, IM3 and IM5 products with an error smaller than 1dB; the errors of the static model are significantly higher.

V. CONCLUSIONS

The new behavioral model can accurately predict nonlinear behavior including compression and intermodulation product generation of an amplifier under fast modulated stimulus conditions. The model is based on a complex envelope representation and can relatively easily be derived by performing a set of pulsed RF envelope measurements on an Agilent t NVNA.

The new model was verified using an NVNA by performing 2-tone measurements in the envelope domain and was shown to outperform a traditional approach based on a static AM-AM AM-PM characteristic. Future work includes validating the model predictions under actual digitally modulated wide bandwidth communication signal stimuli.
REFERENCES


APPENDIX

In this appendix we will show that, with variable “a” being a random variable representing the collection of instantaneous amplitudes $A(t)$ and $E(.)$ representing the expected value operator,

$$\int_0^\infty P_i[A(t-u)] k_i(u) \, du = K_i(\infty) E(P_i(a))$$

under the following conditions:

1. “the input envelope is varying fast compared to the time constants associated with the impulse response $k(.)$"; and we assert the following additional property of the signal:

2. “the statistical properties of the input envelope are stationary";

For simplicity we suppress the subscript “i” denoting the $i^{th}$ hidden variable and we also omit the absolute value sign, but assume then that $A(t)$ is a non-negative real-valued function. We therefore evaluate

$$h(t) = \int_0^\infty P(A(t-u)) k(u) \, du.$$  \hspace{1cm} (18)

We start by splitting the time axis into small intervals with a length $T$. The result is

$$h(t) = \sum_{i=0}^\infty \int_{iT}^{(i+1)T} P(A(t-u)) k(u) \, du.$$  \hspace{1cm} (19)

For any $T$ that is small enough, the value of the function $k(.)$ will be approximately constant over each individual interval. This results in

$$h(t) = \lim_{T \to \infty} \sum_{i=0}^\infty \int_{iT}^{(i+1)T} P(A(t-u)) k(iT) \, du.$$  \hspace{1cm} (20)

The sampled values $k(iT)$ can be taken out of the integral.

$$h(t) = \lim_{T \to \infty} \sum_{i=0}^\infty k(iT) \int_{iT}^{(i+1)T} P(A(t-u)) \, du.$$  \hspace{1cm} (21)

The remaining integrals can be written as the limit of a Riemann sum:

$$h(t) = \lim_{T \to \infty} \sum_{i=0}^\infty k(iT) \left\{ \lim_{M \to \infty} \sum_{j=0}^{M-1} \frac{T}{M} P\left(A\left(t-iT-j \frac{T}{M}\right)\right) \right\}$$  \hspace{1cm} (22)

which can be written as

$$h(t) = \lim_{T \to \infty} \sum_{i=0}^\infty \int_{iT}^{(i+1)T} \frac{T}{M} \frac{1}{M} P\left(A\left(t-iT-j \frac{T}{M}\right)\right)$$  \hspace{1cm} (23)

Consider now the expression denoted by the curly brackets $\{\}$. This expression simply represents the sample average of a large number of sampled values. The statistical law of large numbers states that the value of this average will converge to the expected value as the number of samples increases to infinity. This is mathematically expressed by

$$\lim_{M \to \infty} \sum_{j=0}^{M-1} \frac{1}{M} P\left(A\left(t-iT-j \frac{T}{M}\right)\right) = E(P(a)).$$  \hspace{1cm} (24)

Note that, in deriving (24), one uses the fact that $A(t)$ has stationary statistical properties. Substitution of (24) in (23) results in

$$h(t) = \lim_{T \to \infty} \sum_{i=0}^\infty k(iT) E(P(a)).$$  \hspace{1cm} (25)

The remaining limit corresponds to a limit of a Riemann sum leading to a Riemann integral, and can as such be written as

$$\lim_{T \to 0} \sum_{i=0}^\infty k(iT) T = \int_0^\infty k(u) \, du = K(\infty) - K(0).$$  \hspace{1cm} (26)

As $K(.)$ represents the step response of a low-pass filter characteristic $K(0)$ will be equal to 0, such that

$$h(t) = K(\infty) E(P(a)),$$ Q.E.D.