Abstract:

The demand to put more data through the channel has driven the industry to complex modulations that require low phase noise oscillators and low distortion power amplifiers. While design tools such as link power budgets are excellent for predicting performance at threshold, a different analysis is needed for ‘normal’ receive power levels. Residual bit error rate prediction allows the radio designer to determine the phase noise and linearity requirements for a given quality of service. System bit error rate performance is thus related to key analog metrics that drive cost. The recent growth of the broadband market has created much interest in this little understood topic.
This quiz often helps to create interest in the subject of Residual BER. Imagine you are making an expensive microwave high capacity broadband point to point data link and see what your answers would likely be. [This quiz is often used as the audience settles in for the presentation.]

Now that you have a minute to think about it how many did you answer false? The correct answer is, all are false statements! I hope you all answered correctly, if not don’t worry by the time we finish this paper it should be clear!
This presentation is broken into five parts. First, a brief introduction to the Residual BER problem and QAM Digital Radio for those unfamiliar with the basic technology.

Next, we will review a basic noise and error probability model that is commonly found in many text books. This will serve as a foundation for our Residual BER budget process.

Similarly, a review of common non-linear distortion models is helpful before trying to begin to put the concepts into a composite system. This is followed by a discussion of composite effects.

Finally, an example system budget is presented to tie all the material into a practical form for the system engineer.

Frequently, radio engineers find they have been exposed to all this material but rarely have they seen it applied to the Residual BER budgeting process.
In any digital communications link there are bit senders and bit receivers that are physically separated. These devices are linked together through a network link, often provided by a third party service. The network links can be wire, coaxial cable, fiber optic, microwave or some combination.

One very important measure of the Quality of Service (QoS) of the network link provider is the ratio of bits sent correctly to bits in error. This ratio is called the Bit Error Rate or BER.

Different levels of service quality are required depending on the type of network data being transported between locations. Voice traffic will tolerate much higher error rates than data traffic. Digitized voice can tolerate bit errors as high as 1 bit per thousand bits sent or $10^{-3}$ BER. Computer data demands bit error rates of 1 per million to 1 per trillion or BER's of $10^{-6}$ to $10^{-12}$ depending on content. For example, internet surfing does not demand the same quality of service as bank fund transfers.

BER is an important part of the network operators service offering.
What is “Residual” BER? In a microwave data link bit error rate is a function of the Received Signal Level (RSL) sometimes called Received Signal Strength (RSS).

Very weak signals make many bit errors. The transition from few errors to many at low power is called “Threshold.” A considerable body of knowledge exists on predicting threshold because it is a key factor in determining the maximum physical distance of the data link and its availability to transport data due to atmospherics.
As received signal strength increases, the error rate will fall to a very low level or error floor. This error floor is called the “Residual” bit error rate or “Residual BER”. It is the ‘normal’ operating performance of the data link.

As received power is increased, ultimately the receiver will reach an overload point where the error rate increases quickly.

This presentation focuses on a somewhat different approach for predicting residual BER, from those traditionally used at threshold.
Why is residual BER important? The network provider’s customers demand a certain quality of service based on the type of data payload being carried. The better the quality of service, the bigger the potential market. Thus the network provider uses residual BER as a measure of the quality of the equipment he has purchased.

Unlike threshold and dispersive fade margin, key availability metrics, residual BER characterizes the radio in its ‘normal’ operating received signal strength range. This is the performance the network provider will experience most of the time.

Residual BER measures the combined effect of the digital radio’s modulator, transmitter, receiver and demodulator. It is thus a combined evaluation of the entire link. In essence it is a single measure of a broadband radio link’s quality of service (QoS). Residual BER is a metric defining the service performance similar to the link budget’s fade margin that defines the availability of the service (99.9999% of the time).
Residual BER prediction guarantees that modem and RF will integrate together to deliver consistent error floor performance!

The measurements used to confirm residual BER prediction budgets, allow the technician to separate modulator, transmitter, receiver and demodulator issues, where loop-back and EVM techniques cannot.

Many vendors are now providing products with capacity upgrade paths by increasing the complexity of the radio’s modulation. Residual BER prediction allows manufactures the ability to upgrade modems in the future with confidence that the present RF will support it. As residual BER budgets assure interoperability between different receivers or transmitters!

Residual BER budgets are an essential element for cost optimization of the sources and power amplifier, two of the most expensive pieces of any radio link.

The most important contribution of residual BER prediction is it relates key analog metrics to digital bit errors! This bridges the gap between the network providers quality of service metric and the radio engineer’s analog metrics.
Most modern digital radios use QAM or some form of it. QAM stands for Quadrature Amplitude Modulation which is a vector modulation or Phasor modulation. It is created by taking two vectors which are 90 degrees apart and amplitude modulating them, then summing them together to from a resultant vector. The resultant vector can be modulated in both amplitude and phase.

This vector or “phasor,” can be directed to any number of points which represent a symbol constellation. Typically, the number of points in the constellation is related to a power of 2 ($2^n$) to make digital processing easier.

In the remainder of this paper we will use the 64 QAM symbol constellation for our examples (the techniques are equally applicable to other QAM modulations). Often for simplicity, only a single quadrant of the 64 QAM constellation is shown.
Each symbol represents several bits, allowing more information to be sent with each sample of the vectors position. [Note: Only a signal quadrant of the constellation is shown!]

Sending several bits with each sample has the advantage of decreasing the rate the vector is modulated, thus decreasing the RF bandwidth required to transmit a given amount of information. Decreasing the RF bandwidth requirements provides high spectral efficiency which is often a concern with broadband modulations.

On the receive side the vector is matched up with the symbol that it best fits and bit values are reassigned.

Let’s review the process on the next slide.
In the typical digital radio, bits come to the modulator and are mapped to a symbol point. The vector is then driven to that symbol point.

The signal then is up converted to a high frequency [filtering has been omitted] which radiates easily and where sufficient bandwidth exists to carry the required data rate. The up converted signal is boosted in power with the power amplifier [PA] and directed out through the antenna towards the receiver.

The signal then travels to the receiver suffering attenuation and some distortion from the path.

Upon receiving the signal it is amplified and down converted to a frequency where signal processing is least costly. The demodulator then compares the phase and amplitude of the vector and makes a decision on which symbol best fits, followed by assigning the representative bits.
Ideally, at the receive demodulator a single infinitesimally small sample point would be found in the center of the symbol boundary area. Unfortunately, this never occurs because there is always noise, distortion and interference components present.

Random noise has the effect of creating a distribution of sample points. Phase noise is similar to random noise but is only on the angular axis.

AM/AM distortion causes the symbol point to fall short of the desired point on the radial axis based on vector length. AM/PM distortion causes the symbol point to take on an angular error based on the vector length.

Delay distortion [sometimes call inter symbol interference, ISI] cases the symbol point to be distorted based on the previous symbol point.

Finally, spurious interference will cause the point to take on a circular shape.

These are all unwanted signal impairments which make the symbol decision process imprecise and result in bit errors.
Many of these receiver problems are usually dominated by several key system elements. Generally, they fall in two categories, noise sources and distortion sources.

Local oscillators used in creation and demodulation of the modulation as well as the up and down conversion LO’s are primarily responsible for the phase noise component.

Receiver noise figure is a primary cause of the random noise.

The transmitter power amplifier and receiver first mixer are the most common sources of distortion components, such as AM/AM and AM/PM.

It is interesting to note that these components often represent the majority of the cost of a broadband wireless link (Typically 60% or more of a radio cost is in the sources and power amplifier)! 
Each of these sources of constellation problems has a unique impact on the BER performance. At threshold, receiver noise dominates the BER error mechanisms. At overload, receiver distortion, primarily in the first mixer, dominates the BER error mechanism. The residual error floor is dominated by a combination of the phase noise from all the sources and the PA’s distortion!

The remainder of this presentation will focus on the residual BER floor. It is important to note that the techniques to follow, are equally applicable to overload and threshold. The dominate component(s) of error differ based on the received signal level. Thus, the error versus received signal level curve is really made up of three composite terms (threshold, residual & overload).

Most systems engineers are familiar with threshold calculation (Threshold = \( C/N + NF + BW + KT \), e.g. If: \( C/N = 28\text{dB} \) NF = 5dB, BW= 28MHz, KT = -174.1dBm@273°K, then: Threshold = 28+5.0+74.4-174.1 = -66.7 dBm) which at first glance may not seem related to the techniques described in this paper. The carrier to noise ratio for a given error rate (C/N) embodies the analogous statistical relationship presented in this paper. Noise figure (NF) bandwidth (BW), Boltzman’s Constant (K) and Temperature (T) are used to scale the noise power level to traceable standards.
Now that we have had a brief review of QAM radio let’s examine a common noise and error probability model.
Errors occur when the received phasor sample falls outside the intended symbol boundary. The addition of gaussian noise creates a distribution of sample points about the mean or “ideal” symbol point. If sliced on a single axis the probability density function (pdf) is clearly visible. This distribution is similar to the “bell shaped curve” of test scores.

The pdf area under the curve beyond the symbol boundary represents the probability of that type of error. The probability can be calculated by integrating the area from the symbol boundary to minus infinity.

The central limit theorem can be used to normalized the curve to a Gaussian probability density function where the “standard deviation” (σ) is used to determine the probability of an error. Error probability can then be expressed in terms of the standard deviation of samples. Thus, the primary question becomes how many sigma (σ) are there to the symbol boundary?
Counting the number of standard deviations to the symbol boundary provides a means to determine the probability of a symbol error.

If there is only a single standard deviation to the symbol boundary ($\sigma = 1$), then the probability of that boundary error is $3.5 \times 10^{-1}$ or 35%. If there are seven sigma to the boundary ($\sigma = 7$), then the probability of an error is $2.7 \times 10^{-10}$, which is quite small.

In practice, many higher performance wireless broadband systems compete with the low end of the fiber optic communications. Often these broadband systems are held to the same QoS requirements as the fiber, where residual BER rates of $10^{-12}$ are common place. Raw uncorrected BER rates of $10^{-10}$ are usually sufficient to achieve $10^{-12}$ after correction, thus providing a wireless service of the same quality as that of the fiber. The typical number of sigma to the boundary for a high quality wireless system is usually between 7 and 8.

(Note: The above paragraph is the only place in this presentation where the effects of FEC are discussed. The remainder of the discussion deals with raw uncorrected error rates.)
The preceding discussion was for a simplified random noise model often found in textbooks for the analysis of threshold noise. This model is the basis for threshold effects which are random in all directions and only relevant at low power.

This presentation is concerned with “normal” operating power levels where oscillator phase noise is the dominate source of noise. Local oscillator phase noise is always present and unlike threshold noise is random on the angular axis.

The preceding analysis was simplified to examine only a single type of boundary error. In reality, errors can occur on both symbol boundaries, so a “two tailed” probability model is required. The integration of both “tails” is sometimes seen as a 3 dB factor.
So how does one characterize and account for phase noise in the probability model? First, we must understand that phase noise is a measure of the source’s (local oscillator’s) spectral purity or how perfect the sine wave is produced.

Frequency \( f \) is the rate of change of phase with time. Phase noise is the deviation in phase from the mean rate of phase change (center frequency). Side-band noise power is rarely flat Gaussian with frequency offset and thus it must be integrated to obtain phase noise.

The random nature of side-band noise necessitates the need for a root mean squared (RMS) characterization. Hence, by integrating side-band noise (the difference between the mean power and the side-band power, dBC) in an RMS fashion, phase noise is expressed as an RMS angular error in either degrees or radians.

It is important to note that the limits of integration should start just outside of the carrier recovery tracking loop bandwidth for the lower limit and stop at the symbol rate bandwidth for the upper limit.
A key relationship essential to realize is, that the one sigma distance happens to be identical to the RMS error!!! Often statistics courses are taught in such a way that connection between RMS and sigma are never clearly stated to the engineer.

If there is one element of this presentation to come away with, if you don’t already know it, it is this relationship between sigma and RMS. As we will see, this relationship makes it possible to calculate the probability of a BER from analog metrics!
Integrating side-band noise power over the appropriate limits gives us the RMS angular phase noise in degrees ($\Delta \phi_{RMS}$) that will effect the modulation.

This RMS error ($\Delta \phi_{RMS}$) represents the amount of angular degrees that are contained in one sigma ($\sigma = 1$) of “standard deviation!”

Knowing the angular magnitude of the one sigma and the constellation geometry it is possible to calculate the number of sigma to the symbol boundaries.

Given the number of sigma to the boundary, the normalize pdf yields the probability of that boundary error! Thus it is possible to calculate the probability of a symbol error for each possible boundary error types in the constellation. Thus the effect of oscillator phase noise on residual symbol errors can be calculated!
Having reviewed QAM digital radio concepts, a basic noise model and the extension of that noise model to phase noise, let’s now examine non-linear distortion as it applies to residual BER. As with the noise case, we will start by reviewing common non-linear models and then focus on those which are best for solving the problem of predicting residual BER.
Commonly used linearity metrics are, two tone intermodulation distortion (IMD) testing and amplitude modulation to phase modulation conversion testing (AM/PM). Let’s review each briefly.

Two tone IMD testing is a scalar measurement that is based on the internal mixing of harmonics generated in the device at high powers. It is indirectly related to BER and has long been used as a figure of merit. Unfortunately, correlation studies of BER vs. IMD have shown correlation’s as low as 85%, making system residual BER performance uncertain. Despite the inaccuracies of IMD, historically it has been the traditional test (mostly because of lower equipment costs).

AM/PM testing is a vector measurement typically requiring a vector network analyzer (VNA). It is based on measuring a relative microwave phase shift as power is increased. AM/PM is directly related to the BER mechanism and is an analytical part of the residual BER system budget. Recent decreases in the cost of vector network analyzers now make AM/PM measurement less expensive than IMD measurement and it is gaining popularity as a more modern test.

So which linearity test is best for QAM? AM/PM has the distinct advantage of being the best choice for high QoS systems where residual BER is of importance! Let’s examine why AM/PM is the measurement of choice?
Amplifier Linearity Behavior & Common Metrics

- **Linear Amplification**
- **Saturated Power**
- **Gain Compression**
- **Power at 1 dB of Compression**
- **Two Tone Intermodulation (IMD)**
- **3rd & 5th Order Products**
- **Third Order Intercept Point**

At low power levels amplifiers exhibit linear behavior, such that small signals are amplified by a fixed amount of gain. Given some power input the amplifier is said to behave linearly if the power output is a fixed ratio larger. As the input signal grows larger a point is reached where the output signal will stop getting bigger, the amplifier is said to be saturated and the linear relationship between input and output no longer exists. Many measurements have been devised to characterize this phenomenon.

Gain compression is a term used to describe the difference between the saturating amplifier’s performance and the theoretically ideal performance. So called $P_{1\text{dB}}$ is a measure of the output power at 1 dB of gain compression.

Two tone intermodulation (IMD) is a measurement designed to predict the amount of unwanted modulation energy created by the nonlinear saturation process. It measures the 3rd or 5th order intermodulation products. A third order intercept point can be calculated from the 3rd and 5th order products, which is helpful in predicting the level of intermodulation distortion.

Though commonly used none of these linearity metrics is ideally suited for predicting the residual BER because PA’s are operated in a region where an abrupt change in device linearity occurs where IMD and TOI measurements fail.
To predict residual BER, linearity metrics that directly relate to the QAM vector in amplitude and phase are needed. Gain compression or the difference between the ideal linear gain and the actual gain is amplitude modulation due to amplitude modulation conversion (AM/AM).

As power is increased the phase delay through an amplifier begins to change as it nears saturation. This change in phase shift as the power is increased or modulated is AM/PM modulation. As we will see later, this modulation is additive to the QAM vector.

AM/AM and AM/PM are unwanted modulations that effect the accuracy of where the symbol point position on the constellation falls.

The two graphs show the distortions in relation to each other (i.e., same power scale). Note that significant phase shifts occur before significant amplitude shifts. Typically only a few tenths of a degree of AM/AM occur when several degrees of AM/PM have built up. The QAM modulation is much more “sensitive” to the AM/PM distortion and a few degrees of distortion are quite significant, where as 0.1 to 0.3 dB of AM/AM has little effect. This phenomena allows a key simplifying assumption that AM/AM is a secondary effect and can be ignored in our analysis.
Sometimes it is helpful to review the mechanisms that actually generate these unwanted distortions in the power amplifier. Let’s use a simple GaAs MESFET model to illustrate the principles. To keep it simple we have omitted the effects of microwave matching circuits that are of finite bandwidth and linear in nature.

The GaAs FET voltage to current characteristic behaves as a square law device. If driven with a small signal input voltage, then current is modulated about the $Q$ point on the output. Typically, most QAM microwave power amplifiers are built as “class A” designs for maximum linearity.

The raw DC power supplied to the FET to establish the $Q$ point must be limited to constrain the devices operating temperature. Constraining the devices operating temperature slows the semiconductors defect migration to assure a long operating life.

Next, let’s see how IMD products, AM/AM and AM/PM are created in the device.
In a two tone IMD test, identical amplitude tones separated slightly in frequency are injected into the input of the amplifier. The power supply cannot deliver additional current so the superimposed sine waves are “clipped.” This gives rise to 2nd, 3rd etc. harmonics which mix together forming the IMD products.

This mixing action actually takes place in the junction of the device and often the harmonics are substantially attenuated by the band limited output matching networks. Thus it is often not possible to directly compute the intermodulation products using the measured harmonic power for microwave amplifiers. It is however a simple process to measure the relative attenuation of the IMD products with a spectrum analyzer.

Does saturation really produce such an abrupt clipping of the two sine waves? Well, that depends very much on the particular FET and how it is biased. Forward gate rectification can be very abrupt, whereas pinch off or the square law curve are much more subtle saturation characteristics. Hence the amount of harmonic energy available to be converted into intermodulation products is very dependent on the abruptness of the non-linearity! Here lies the major problem with using IMD to predict BER from correlation studies. The IMD measurement is just a single point on the $\frac{P_{\text{Output}}}{P_{\text{Input}}}$ curve and only predicts linearity for small signal characteristics, but QAM modulation operates over a range of vector amplitudes near saturation that include abrupt changes in linearity.
The AM/AM mechanism is very simple to examine with our amplifier model. If we put a single sine wave into the amplifier and the power source cannot supply the necessary current resulting in clipping the amplitude of the output is reduced. This reduction is gain compression or AM/AM.

AM/AM is easily measured with either a source and spectrum analyzer or a network analyzer.
The phenomena that gives rise to AM/PM again begins with the power source limitation creating a clipping or “mushing” of the waveform. Since the top of the waveform is not correctly amplified, the average value or “zero crossing” is offset from the original position. This offset in “zero crossing” occurs where the sine wave has finite slope creating a phase shift in the output signal.

It is interesting to note if the input signal is increased still further, ultimately clipping on the bottom of the sine wave would occur as the output begins to resemble a square wave. Clipping on both ends causes the average offset to come back towards that of the original sine wave; Hence, the characteristic rise and fall of the AM/PM curve.

It is also important to note that the microwave matching circuits strongly effect the impedance, hence the voltage and current relationship as well as the clipping on the output of the device is effected.
The AM/PM phase shift as a function of signal amplitude or vector length distorts the ideal symbol location of the QAM symbol constellation.

Outer symbols have the largest vector length and suffer the most AM/PM distortion.

AM/PM testing is typically done with a CW sine wave (though it possible to measure it with a modulated signal and Vector Signal Analyzer, VSA). This is an important fact because it relates a CW parametric test to the actual distortion impairment of the QAM symbol constellation!

The ability to relate a parametric analog test to the actual error mechanism provides the means to predict digital error rates from traceable standards.
So how does one relate the vector length to known traceable standards? And what range of vector lengths do we need to measure the distortion over?

What do the power meters we used to setup our transmit power actually measure? A power meter measures the average CW or modulated power. Testing AM/PM with a CW source requires relating the average modulated power to the average CW power.

At first glance, this can be done by using the constellation geometry to calculate the average vector length. But, the worst case AM/PM occurs at the longest vector length or peak power. Calculating the peak to average ratio from the constellation to correct the measured average power is an important step for determining what CW test signal power is necessary for AM/PM testing.

A peak to average power correction is only one part of the determining the highest power to test AM/PM at. In between symbol states the vector overshoots the boundaries of the constellation. The so called “overshoot power” or “trajectory power” represents the longest vector length. Though understood, the overshoot phenomena is beyond the scope of this paper, so for our purposes let’s simply say it is a function of the base band filtering $\alpha$. 
What test conditions are necessary to properly characterize the distortion of the power amplifier? The gain and phase distortions need to be characterized over the range of modulation powers. This means that AM/PM should be tested from the smallest vector needed to produce the modulation, to the largest vector or in other words, over the “vector range.”

Correction factors for peak to average and overshoot powers, typically about 3 dB each, must be added to the average power to come up with the overshoot power. Similarly, minimum to average power can be subtracted from the average power to determine the lowest power required. In practical terms the low power limit can be set to where no appreciable AM/PM modulation is observed, eliminating the need to calculate minimum to average.

The important thing is that we actually measure the amplifiers linearity at the overshoot power where the distortion will be the most significant. Testing at average power (what the power meter measures) would be very misleading.

One approach to determining the appropriate vector range is to measure the complementary cumulative density function of the modulated signal. This vector signal analyzer (VSA) test shows the overshoot or trajectory vector length.
There are two types of AM/PM measurements, spot and swept. The classical diagram of AM/PM or $\Delta \phi$ versus power level, often seen in linearizer work, is a spot frequency versus swept power measurement. This measurement is best suited for fixed frequency operation since the matching, hence AM/PM, is usually a function of frequency.

Another approach is to use a swept frequency measurement at a spot power delta. This measurement is best suited for broadband devices that operate over a range of frequencies. It does require the assumption that the AM/PM increases monotonically over the power range of interest which is virtually always the case for QAM signals.

Swept frequency AM/PM measurement requires the PA output power to be calibrated across the band of interest at a low power (power cal). Next, the phase is calibrated to zero with the PA in place (since this is a relative measurement). Finally, the power is increased to the overshoot power and the AM/PM across the band can be observed.

Practically, AM/PM measurement must be done quickly to avoid junction cooling effects that influence the accuracy of the measurement. If it is not possible to make the measurement quickly the more complicated complex stimulus/response method is required.
Now that we have reviewed some of the basic principles behind QAM digital radio, a phase noise probability model and non-linear elements, let’s review some of the mathematics necessary to combine them as well as the assumptions and approximations involved.
Distortion is directly additive to the noise because it operates on the vector itself. This has the effect of offsetting the mean value of the probability density function.

Distortion by itself has no probability of an error! At first this might seem counter intuitive but it requires the randomness of phase noise to create random dribbling errors. What if the distortion was so large that sample point fell beyond the symbol boundary? It would make an error, but without noise it would always make the same error in a deterministic way (non-varying BER). This just doesn’t happen because there is always some noise present.

Residual BER is a function of BOTH the power amplifier linearity and the phase noise of all the sources in the system! This is a very important point, for often there is disagreement over whether the PA or one of the sources is to blame for dribbling errors. The answer is that they both influence the error floor and only a judicious allocation budget (usually based on implementation cost) can sort out which element is bringing the system down.
System phase noise can be represented by a noise vector which adds geometrically to the desired modulation vector. Noise from more than one source can be added geometrically to obtain the total noise for the system.

The sources in the modem and the conversion process all contribute to the overall system phase noise. Thus, residual BER is a function of BOTH the modem and RF sources. The residual BER is affected by BOTH transmitter and receiver sources! These are very important points! It is not possible to evaluate the system residual BER without both the modem and the RF. Likewise it is not possible to evaluate the system residual BER without both the modulator/transmitter and receiver/demodulator.

This means that loop back testing to exonerate the modem from dribbling error problems is not a valid approach. Likewise, most golden transmitters and golden receivers are not valid for testing residual BER. Only golden units with sources and amplifiers that have the worst case phase noise and distortion can be used for testing residual BER (a difficult proposition to construct).

High quality of service systems require parametric testing of the primary phase noise and distortion components to guarantee consistent interchangeable part performance.
The QAM symbol constellation has some important geometric effects to consider. The outer symbol points can tolerate the least angular error. In the 64QAM constellation the outer most point can tolerate a maximum of 7.7° of error before making a symbol decision error versus the inner most point which can tolerate 45° of error. The outer most point also has the longest vector length and corresponding highest angular distortion.

The maximum angular error is symbol location dependent, but the phase noise is a constant for all symbol points. Thus, the number of sigma to the boundary is dependent on the symbol location, with the outer symbols having the fewest sigma to the boundary.

Error probability of each symbol must be weighted based on the probability of symbol occurrence. Usually, symbols are equi-probable and symbol boundaries are setup on a simple grid pattern. There are some QAM modulations where probability of occurrence and grid patterns are not so simple (usually to take advantage of the fact most errors are made on the edge of the constellation) and this must be taken into account in the system model.
In developing our model of residual BER we have focused primarily on the source phase noise and the power amplifier AM/PM distortion. These key components usually represent the vast majority of the residual error budget and are often responsible for the vast majority of the cost of the radio. There are other secondary contributors to the error floor to be aware of, such as group delay distortion or inter symbol interference (ISI).

In most cases group delay distortion is not a significant factor largely because of the tremendous power of modern digital equalizers. It is useful to be aware of it’s effects to minimize the equalization required for static filter effects and maximize the equalizers performance for dynamic channel fading. In some burst systems where equalizer performance is limited, group delay distortion should be taken into account.

Finally, before briefly covering group delay distortion, be aware there are many other secondary contributors to the symbol error mechanisms. In our simple model we will assume they are insignificant, which in practice is usually the case.
Filters introduce group delay into the modulated channel. The spectral energy associated with a change in phase is dependent on the magnitude of phase modulation. Small changes in phase occupy small bandwidths, large changes in phase occupy larger bandwidths.

The delay difference associated with the two spectrums produces a phase error on the modulation vector. This error is based on what the previous symbol sequence was and is another way of looking at inter symbol interference (ISI).

Fortunately, for most modern digitally equalized radio’s the equalizer reduces this error to an insignificant portion of the residual BER budget and it can be ignored.
There are many approaches to measuring the different error mechanism types. The most common solutions are shown on this slide, and of course Agilent offers a complete portfolio of equipment.

Among the most popular test approaches are noise figure analyzers (NFA) for random noise characterization, spectrum analyzers (SA) for phase noise integration (high performance applications require phase noise test sets) and vector network analyzers (VNA) for AM/AM and AM/PM measurement.

Delay distortion can be measured either with a vector network analyzer (VNA) or a vector signal analyzer (VSA). Vector signal analyzers (VSA) also excel at identifying spurious interference with tools like error vector spectrum.
At this point we might ask: Why not use error vector magnitude (EVM) measurements to estimate BER versus separate AM/PM and phase noise measurements?

Let’s examine two scenarios, one with a lot of phase noise and a little distortion and one with little phase noise and lots of distortion. Which has the lower BER? Which has the lower EVM? The one which has the least number of sigma to the boundary will have the highest BER, hence the high phase noise example will have high BER. If we add up the RMS phase noise and distortion components both scenarios have 6°! They have the same EVM!
EVM is a summation of effects. It does not differentiate between random effects which add geometrically and possess probability densities, from distortion effects which add directly and are deterministic. EVM is useful because its characteristics can tell us something about the nature of the problem or where the signal was degraded, but it cannot be directly related to BER, the acid test for the network operator. EVM must be decomposed into the different types of error mechanisms, so the appropriate mathematics can be applied to relate those components to BER. However, the characteristics of the error vectors themselves to the trained eye, can provide tremendous “qualitative” insight when it comes to diagnosing problems.
To try and pull all these concepts together into practical technique, let’s review an example residual BER system budget. The goal is to predict the residual BER of a 64 QAM radio design buy assignment of phase noise and AM/PM performance.
The first step in predicting residual BER is to calculate the symbol vector lengths for every point in the constellation. Using symmetry simplifies the work by only requiring the computations for a single quadrant.

Second, the maximum possible phase error for each symbol is calculated.

Third, the phase noise and distortion components are allocated on a trial basis.

Fourth, the symbol error probability and BER are calculated from the normalized probability density function. If the results are unacceptable, reallocation of the phase noise and distortion components must be repeated until the desired results are achieved.

Finally, once the desired residual BER has been achieved the phase noise and distortion allocations must be subdivided across the system.

Now that we have a procedure let’s see how it works with some real numbers.
First, using Pythagoras’s theorem calculate the magnitude of each symbol vector in the constellation. Using symmetry the work can be simplified by only calculating a single quadrant.

The peak symbol magnitude or in some cases magnitudes (128 QAM) can be determined.

Summing up each vector and dividing by the total number of vectors gives the average vector length.

The peak-to-average ratio can be calculated by dividing the peak symbol vector by the average symbol amplitude. The peak-to-average ratio is usually expressed as a twenty log ratio so the power variation due to the constellation geometry can be assessed. The peak-to-average ratio expressed in dB is also used in conjunction with the trajectory overshoot to determine the correct maximum power to test AM/PM conversion.
Using the constellation geometry, the maximum permissible phase error to the symbol boundary can be calculated. Begin by calculating the angle to the symbol point ($\phi_{iq}$) using an arc cosine relationship and realizing that the symbol vector length previously calculated is the hypotenuse of the triangle.

Similarly, the angle to the symbol boundaries ($\phi_{B1iq}$ & $\phi_{B2iq}$) can be calculated by adding or subtracting the distance from the point to the boundary and then using the arc cosine relationship. Two equations are necessary depending on where the symbol point is located in the constellation. Symbol boundaries are intersected either on the horizontal axis, vertical axis or on both axis (diagonal) with angular rotation of the vector.

Subtracting the symbol point angle from the boundary angle gives the maximum angular error permissible ($\Delta \phi_{Max}$) before an incorrect symbol is detected.

An important simplification can be made by assuming that the clockwise and counter clockwise maximum angular error ($\Delta \phi_{Max}$) is the same. Though these angles differ slightly, in practice the small difference can easily be neglected.
The geometry of the constellation locks down the maximum angular error ($\Delta \phi_{\text{Max}}$). The angular error is composed of a distortion component and some number of sigma times the phase noise component (neglecting other possible error sources, generally a good assumption). Next, an allocation of the distortion component and phase noise component must be made.

Properly allocating between distortion and phase noise can have a significant impact on the overall cost of the system! This can’t be understated, for these allocations affect the performance requirements of the sources and power amplifier, the most costly parts of the radio (typically 60% of the overall cost)! This is where true systems expertise and a sound budget really pays off.

Though it is beyond the scope of this paper to delve into the economics of the allocation and this topic should be relegated to the experienced system engineer, a couple of guidelines maybe helpful. High QoS systems requiring “fiber-like,” error rates generally require 7 to 8 sigma making the total phase noise requirement around 10% of the maximum angular error. The distortion allocation is usually lowest cost just below 50% of the total maximum angular error. As frequency increases the $20\log(N)$ multiplication of phase noise, favors more of the budget being allocated to phase noise. Using these considerations for our model we will assume 3.000° of AM/PM distortion and 0.600° of total phase noise.
The previous equation can be solved for sigma (σ_{iq}) for each of the constellation points. It can easily be seen that the number of sigma to the boundary is much lower for the outer states than for the ones close to the origin. This means that virtually all the residual errors occur in the outer most symbol points.

In the above table we have made an important simplifying assumption. We have assumed that the distortion and noise components are symmetric about center of the symbol point and have neglected to account for the differing number of sigma to the boundary due to the fact that AM/PM has the effect of offsetting the mean in a single direction. At first glance this may seem like a gross approximation, but in practice it results in relatively reasonable errors for a couple of reasons. In the case where there is almost no distortion, true symmetry does exist. In the case where distortion is large, a single tail of the pdf dominates and our model is off by a factor of two. A factor of two may seem large but usually when we are evaluating BER the concern is the exponent, not the precise number (i.e. rarely do we care if it is 1.0 x 10^{-12} or 2.0 x 10^{-12}, more likely we care if it is 10^{-12} versus 10^{-8}). If your application requires that extra precision it is necessary to have two separate tables, one which account for clockwise boundary errors and another which accounts for counter clockwise errors.
The number of sigma’s to the boundary enable the use of the normalized probability density function (pdf) to calculate the probability of each symbol error ($P_{iq}$).

The integral of the pdf is difficult, but fortunately there are both tables and spread sheet functions such as NORMDIST($X$, $\mu$, $\sigma$, $C$) which make it very easy to solve.

Once the individual symbol probabilities have been calculated, they can be averaged to yield the probability of a symbol error. In this example the probability of a symbol error is approximately $1.7 \times 10^{-13}$. This is quite good and on par with fiber optic performance. It is also important to note that this is before FEC where an additional one or two orders of magnitude improvement could be obtained.
The symbol error probability must now be converted to the bit error rate (BER) or probability of a bit error. The conversion of symbol errors to bit errors is bit mapping dependent (i.e. the arbitrary assignment of six different bits to each symbol in the constellation).

Generally speaking, when mapping the bits to symbols, it is important to try to make adjacent symbols differ by as few bits as possible. Ideally, only a single bit differs in the adjacent symbols and a symbol error creates only a single bit error (there is no error multiplication).

The conversion factor between symbol errors and bit errors is typically a low number. Again, the focus is usually on trying to access the order of magnitude rather than the specific number for high QoS systems. Thus in practice with most bit mappings it is usually a close approximation to assume a symbol error results in a single bit error! The symbol error probability calculated is usually a close approximation the BER.
If the calculated BER is acceptable no reallocation of distortion or phase noise is necessary. The final step is to subdivide the total allocation into the components of the system which generate it.

In the case of phase noise, our system has six different sources which must total up to the 0.600° allocated. Recalling that noise adds in a RMS fashion, each of the sources integrated phase noise is geometrically summed to add up to the 0.600° allocated. Again the systems engineer chooses the phase noise requirement of each source to minimize cost. The key consideration is the $20\log(N)$ relationship of multiplied noise (i.e. high frequency sources tend to have more noise than low frequency sources).

A similar subdivision of the distortion budget can be made, but to a lesser extent. It is generally wise to reserve approximately 10% of the distortion budget for secondary effects such as incompletely compensated group delay, PA power leveling, etc..

The skill of the systems engineer in making the right allocations and subdividing the budget into practically realizable values affects the cost and competitiveness of the radio system significantly!
We now revisit the quiz presented at the beginning of the presentation. A radio “dribbling” errors (small numbers of errors scattered across time, versus a large single burst of many contiguous errors) at normal received signal levels is a classic sign of a residual BER problem.

The technician is often convinced in situations like this that because the problem follows a specific PA, that PA must be the cause, but often it is not! The residual BER is a function of all the phase noise sources as well as the distortion sources. It is the total which matters, thus the only way to identify which element is at fault is to establish a worst case maximum budget for each contributor and then test them independently!

The modem loop back test is of little value for residual BER because the major contributors to the residual error floor (PA, up/down conversion sources) are removed hence a very unrealistic evaluation of system performance.

Let’s look at a realistic example. . .
The table shows the data from three example links.

In the example, T/R Link shown in Case #1 exceeds the maximum angular error, causing it to dribble errors at an unacceptable rate. Again the, technicians suspect the power amplifier in situations like this. Indeed, if the PA of Case #1 is swapped with PA of Case #2, a radio that is working fine, Link #2 will begin to dribble errors (note most of the numbers are the same). Similarly, the Link of Case #3, which starts out okay will also begin to dribble errors when the tech. swaps the PA from Case #1 for PA of Case #3. The technician is now convinced in situations like this that because the problem followed the “bad” PA, it must be the cause. As a result a $3000 PA gets put to the side as scrap!

But are all those PA’s really unusable? Actually, they may be fine. Let’s see why . . .
Now we have constructed a system Residual BER budget Spec. [shown on the left]. Notice if the crystal oscillator from the demodulator in Case #1 had met its spec., then Link #1 would perform fine with the original PA [#1].

The author has encountered this exact case in two different companies where the cost of the modem source was ≈$10 versus the ≈$3000 for the PA. Said another way, improvements in the PA would cost hundreds or thousands of dollars to make versus improvements in the demodulator source which would cost pennies!
Many vendors use IMD and phase noise to predict if a QAM radio will dribble errors. This approach is purely empirical. IMD is also a spot power characterization and doesn’t account for the effects of changing compression characteristics as a function of power over the vector range. Worse yet, IMD is often applied just below saturation where its prediction utility is of little value (clipping begins to dominate versus square law curves). While it may seem easy to measure error rates at different power levels and correlate them with IMD measurements, unfortunately, the correlation coefficient can be as bad as 85% for 128QAM! Said another way, 15 out of every 100 radios would fail their residual error count! AM/PM (which is most accurate when clipping begins) and phase noise being analytically related to the BER can calculate the error rate exactly.

Often times dribbling errors focus attention on the PA because it is usually the easiest component to change performance by reducing the transmit power. The PA then “seems” to have the greatest effect on residual BER.

Note however, the integrated phase noise contribution is multiplied by the number of sigma (σ) required for an acceptable error rate (usually between 7-8). Thus, changes in phase noise have a much greater impact than changes in distortion. Unfortunately, phase noise usually cannot be adjusted like PA output power, so this sensitivity is rarely observed with practical hardware.
The residual BER budgeting process is a key analysis in producing a cost effective wireless data link that meets the quality of service requirements demanded by the network provider. Using a combination of phase noise and AM/PM distortion measurements, the sources and PA can be characterized to deliver consistent combined residual BER performance.
So remember. Do **NOT** use Golden modems as part of your production process when you measure Residual BER. The “Golden” modem, with its better than normal performance can easily mask other problems in your radio chain, causing a unit to pass in production that dribbles errors during operation in the field!
This paper has covered the process for predicting the residual BER from the analog metrics which influence it. Accurate characterization of these metrics is an essential part of providing the most system for the money and Agilent has a complete line of products to assist you in developing and optimizing your design as well as controlling it in production!