APPLICATION NOTE

Calculation of CPW Resonator Properties Using the Quantum Technology File

Extraction of resonance frequency, capacitance, and inductance

KEYSIGHT

Introduction

Here we review a method of calculating the main characteristics of a coplanar waveguide (CPW) transmission lines using the given technology file for superconducting quantum computing chips. The technology is based on a metal layer (e.g., aluminum) deposited on a semiconductor layer (e.g., silicon). Additional layers are airbridge, via for airbridge, and air.

The readout resonators of a superconducting qubit are based on the CPW transmission lines, and they can be a quarter or a half wavelength depending on the design. We assume a quarter wavelength resonator is the design goal. The same CPW can be used in designing common feedlines (quantum buses) and design of Purcel filters e.g., the ones based on half wavelength resonator.

Setting Up the Technology File

The process of opening Keysight ADS, creating a new workspace or opening an already existing workspace is the same as explained e.g. in the demonstration video at Keysight Technologies **Learn** or the **Examples** documentation for **Single Superconducting Qubit** included in Keysight ADS (See Appendix A). After executing the Keysight ADS and creating a new workspace on your preferred path, you are asked to select the technology for the library of workspace. Figure 1 shows the list of available technologies from which you choose **Quantum Technology**.

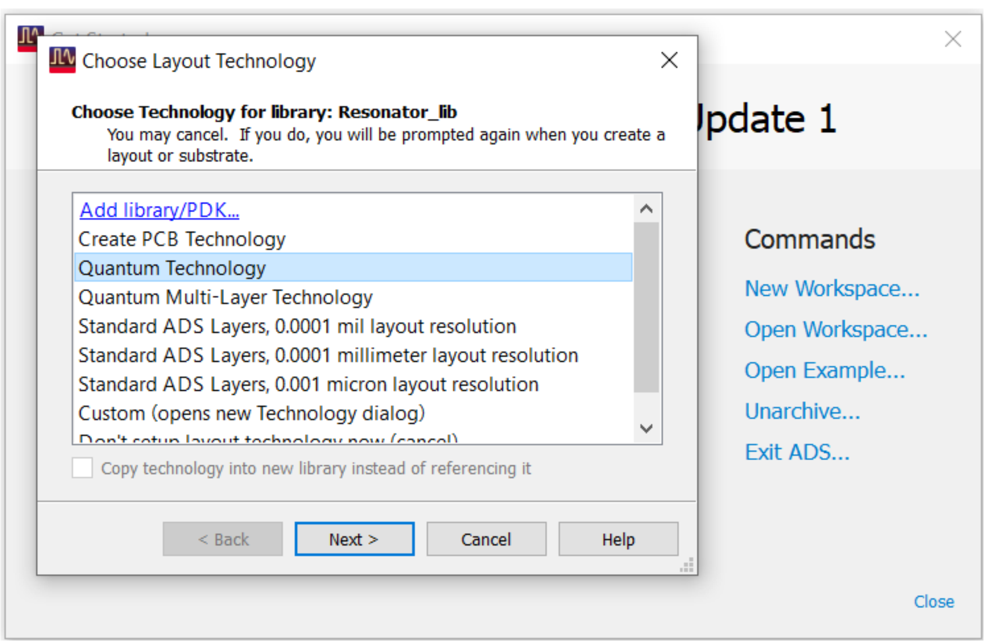


Figure 1. Choosing the technology for library after creating a workspace.

Thereafter you are asked to define or (**Set Up**) the layer thickness and type of material assigned to each layer as shown in Figure 2. After pressing the (**Finish**) button the technology appears under the workspace path (in the main menu of Keysight ADS) as **tech:subst** icon. See Figure 3.

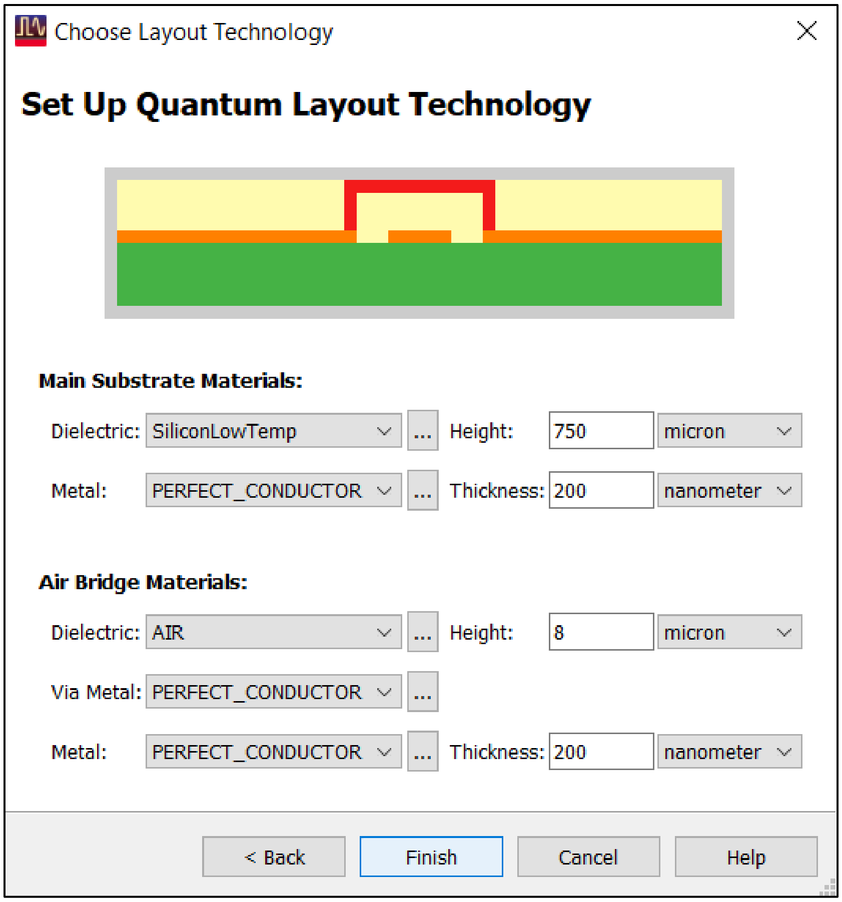


Figure 2. Setting up the layer materials and thickness (or height).

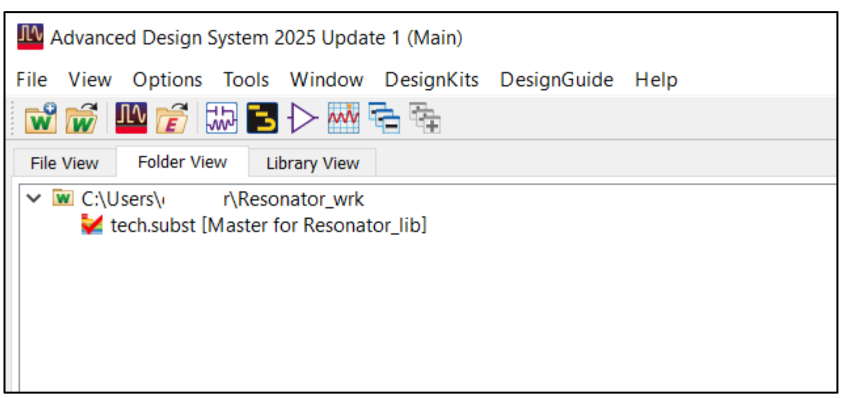


Figure 3. The main menu of ADS shows the **tech:subst** is added to the workspace under your favorite path. Here the workspace is named **Resonator_wrk**.

By clicking on the **tech:subst** a window is opened as shown in Figure 4 in which you can assign different materials to each layer and change the thickness or material property in the **Stackup** table if need be. For more details refer to the **Examples** documentation for **Modeling of Superconducting Resonator with Kinetic Inductance** included in Keysight ADS (See Appendix A).

Controlled Impedance Line Designer (CILD)

On the top menu of **tech** window in Figure 4 you see two icons. Alternatively, you can select the **Tool** and see the opened menu in which there are **Controlled Impedance Line Designer** (CILD) and **Via Designer**.

The controlled impedance line designer helps you to design different kinds of transmission lines using the layers available in your technology of choice. Also it helps in optimizing the design to reach a desired impedance value, sweeping the analysis, and performing a statistical analysis due to statistical variations of geometrical line parameters e.g., width, gap, thicknesses.

By clicking on the cild icon (the one with **Z=50** on it) on the top menu in **tech** window, a new window as shown in Figure 5 appears. We are going to design a single ended (not differential or edge coupled) CPW with 50 Ohm impedance using the main perfect conductor (metal) layer on silicon substrate. It is assumed that the layer on top of the metal layer is air. On the left side of the cild panel select the **Type** to be **Coplanar Single-Ended**. Then you the default layer names appear on the three selectable lines under the **Type** bar. Now you must change them to make the CPW you want.

For this example, we change the **Top Plane** to **<None>**, the **Signal** to **cond**, and **Bottom Plane** to **<None>**. Then the CPW is formed on the silicon substrate (see the yellow-colored cross sections in Figure 5). On the right section of the cild panel (under **Variables**) select the typical frequency (**freq**) (e.g., 10 GHz), **Length**, **Width,** and **Clearance** of the CPW. Width and Clerance are the same as CPW width (W) and gap (g) or spacing (s) in other textbooks. Refer to Pozar's or Collin's microwave engineering textbooks for the definitions [1-2]. We start from (g, W, g) = (9 μm,15 μm, 9 μm) for the CPW and see how cild calculates the characteristics of this line. Note that the **Length** here does not matter as the length of the resonator or feedline will be decided after finding out the effective dielectric constant. On the bottom right section of cild panel you see two tables under **Electrical** which are still empty because you have not yet run the simulation. These empty tables are electrical properties of the transmission line (here CPW): **TML properties** and **RGCL**. The analysis is run by pressing the run icon (gear-shaped icon) on the top menu (above Analysis) and the results are reported in the bottom-right tables.

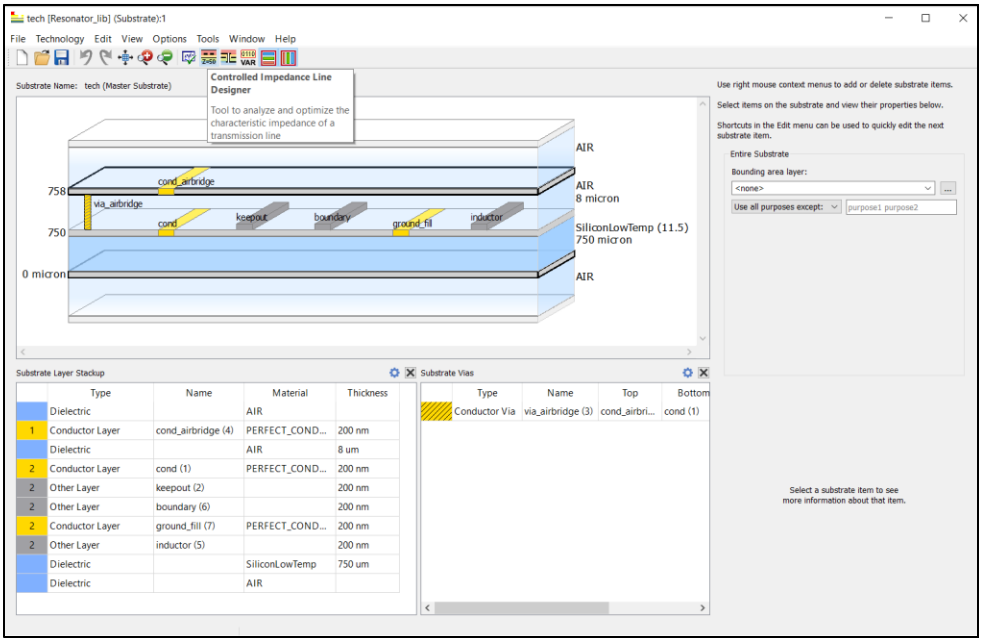


Figure 4. The **stack up** of substrate layers in the chosen technology. The **Controlled Impedance Line Designer** is the icon with Z= 50 on it or alternatively it can be found under the **Tool** on the top menu.

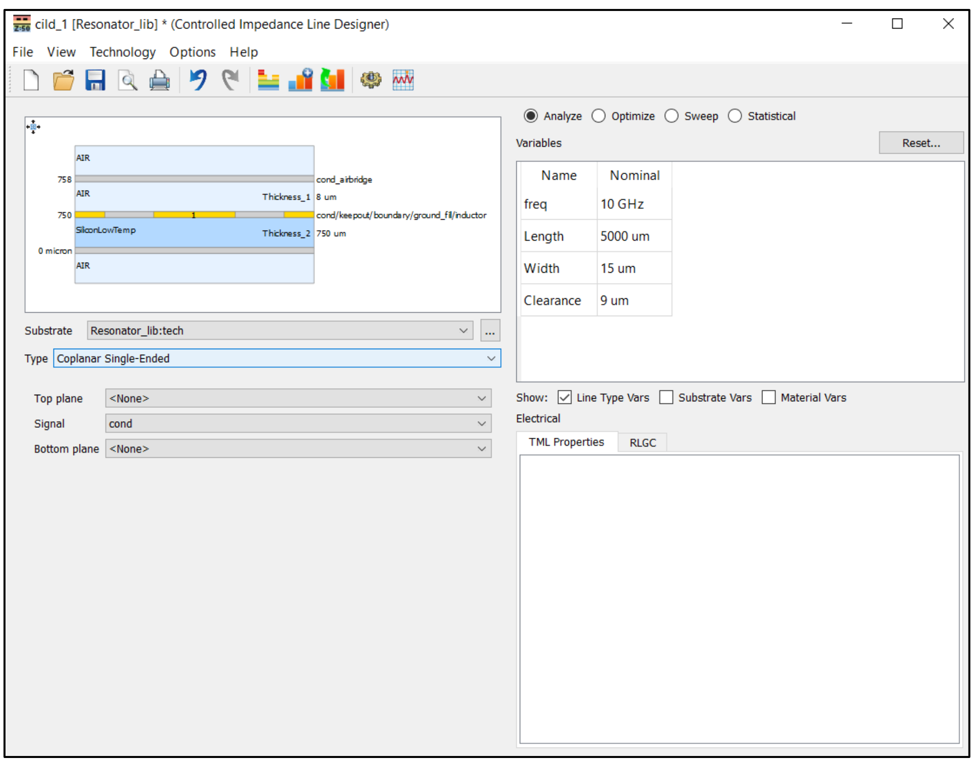


Figure 5. The **Controlled Impedance Line Designer** (cild). The pop-down menus under **Substrate** (**Type, Top Plane, Singal, Bottom Plane**) are used to design the desired transmission line cross section (e.g., the yellow-colored CPW cross section) using the **Width** and **Clearance** given in the top-right table under **Variables**.

Calculation of L, C, and Resonance Frequency

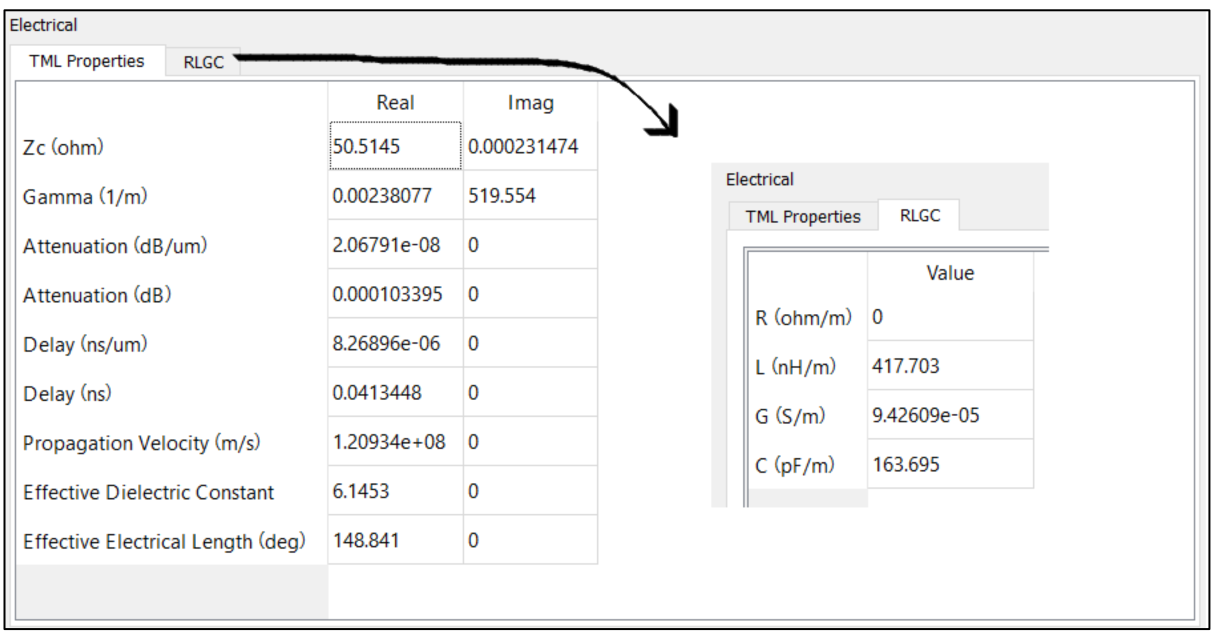
After running the simulation, the results are reported in **TML Properties** and **RGLC** (see Figure 6). Transmission line properties (**TML Properties**) include impedance, effective dielectric constant, loss etc. **RGLC**, as its name suggests, reports the resistance, conductance, inductance and capacitance of TML per unit length. The schematics of such a so-called unit cell of a transmission line is shown in Figure 7. As can be seen in Figure 6, the impedance is close to 50 Ohm (**50.5145 Ohm**) and the effective dielectric constant is $\varepsilon_{eff} = 6.1453$. Note that in the technology file silicon is assumed to be low loss with loss tangent (10^{-5}) and dielectric constant 11.5 (for cryogenic temperatures) and the conductors are perfect conductors (**pec**). You can check these data by clicking on the **Technology** option on the top menu of tech window and choose (**Material Definitions…**).

The inductance and capacitance per unit length are approximately L = 417.7 nH/m and C=163.7 pF/m, respectively.

The length of the quarter wavelength resonator: we use the effective dielectric constant given here to calculate the length of the resonators for this technology and CPW type. Let's assume the desired qubit readout resonator frequency is $f = 6\,GHz$, then the required length is found from the following

$$l = \frac{\lambda}{4} = \frac{v}{4f} = \frac{1}{4f}\frac{c}{\sqrt{\varepsilon_{eff}}} = \frac{3 \times 10^8 mHz}{4 \times 6 \times 10^9 Hz\sqrt{6.1453}} = 5042.4\ \mu m$$

Where v is the velocity of the wave in transmission line and it is found from $v = c/\sqrt{\varepsilon_{eff}}$, and c is velocity of electromagnetic wave (light) in vacuum.



Electrical — TML Properties

	Real	Imag
Zc (ohm)	50.5145	0.000231474
Gamma (1/m)	0.00238077	519.554
Attenuation (dB/um)	2.06791e-08	0
Attenuation (dB)	0.000103395	0
Delay (ns/um)	8.26896e-06	0
Delay (ns)	0.0413448	0
Propagation Velocity (m/s)	1.20934e+08	0
Effective Dielectric Constant	6.1453	0
Effective Electrical Length (deg)	148.841	0

Electrical — RLGC

	Value
R (ohm/m)	0
L (nH/m)	417.703
G (S/m)	9.42609e-05
C (pF/m)	163.695

Figure 6. The electrical characteristics of (g, W, g) = (9 µm,15 µm, 9 µm) CPW line.

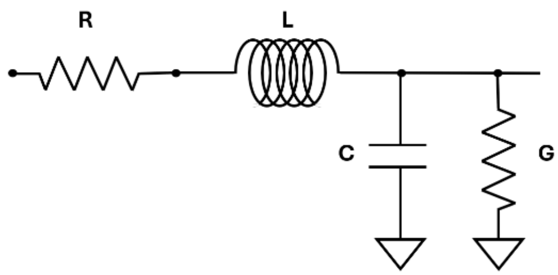


Figure 7. The unit cell of a transmission line and the circuit parameters per unit length.

The total capacitance and inductance of the resonator are found from **RGLC** table as follows,

$$C_{total} = Cl = 163.7\frac{pF}{m} \times\ 5042.4\ \mu m =\ 0.8254\ pF$$

$$L_{total} = Ll = 417.7\frac{nH}{m} \times\ 5042.4\ \mu m = 2.106\ nH$$

From this it is instructive to recheck the resonance frequency of the resonators. The velocity of EM wave in the TML is also found from the capacitance and inductance per unit length (L, C) using $v = 1/\sqrt{LC}$,

$$f = \frac{v}{\lambda} = \frac{1}{4l\sqrt{CL}} = \frac{1}{4\sqrt{Cl\ Ll}} = \frac{1}{4\sqrt{C_{total}L_{total}}} = 5.9962\ GHz$$

This is very close to the desired $f\ =\ 6\ GHz$ design value. Note that in the above, we wrote $\lambda = 4l$, as the resonator is a quarter wavelength. The above equation must be updated for a half wavelength resonator using $\lambda = 2l$.

You can also use the **Optimize** option on the top right menu in cild window to reach the impedance you want (see Figure 8). For example, you can set Z_c = 50 Ohm, keep W = 10 µm and press **Optimize** button for **Clearance**. Then the optimizer tries to find you a value for **Clearance** which gives you 50 Ohm. Here the value is found to be g = 8.48324 µm instead of g = 9 µm. Of course, achieving this value with all the digits in fabrication (considering etching errors and unevenness of edges) is not possible and there will be always tolerances and deviations for the ideality.

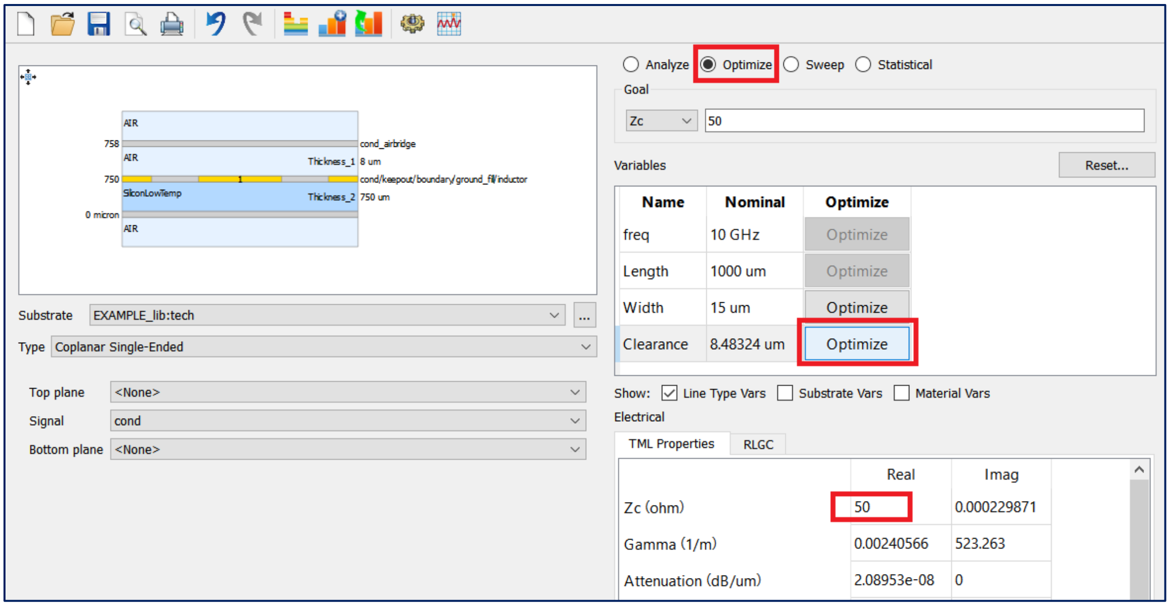


Figure 8. Using **Optimize** to obtain a **Clearance** which gives an exact value of 50 Ohm. Compare this new value with the g = 9 µm for which we obtained 50.5145 Ohm.

We continue with (g, W, g) = (9 µm,15 µm, 9 µm) for the rest of the calculations in this application note.

Caveat

Sometimes users wonder why don't we use the familiar formula, $f = 1/2\pi\sqrt{C_{res}L_{res}}$ for resonance frequency here? Also, some thesis and papers show this formula as resonance frequency even when the context is about distributed CPW resonators!! This formula is for a lumped resonator not a distributed one like a quarter wavelength resonator. I intentionally wrote C_{res} and L_{res} in $f = 1/2\pi\sqrt{C_{res}L_{res}}$ to emphasize that these values are not the same as C, L and C_{total} and L_{total} which we found above. If you insist on using the lumped formula for the resonance frequency, then you must calculate the DC equivalent values of the capacitance and inductance from the distributed CPW resonator characteristics i.e., C and L. Chapter 6 of D. Pozar's book [1] calculates the lumped capacitance and inductance of a quarter-wavelength resonator (with a shorted end) by equating the resonator to a parallel lumped RLC resonator in the vicinity of the resonance frequency (ω_o).

Near the resonance frequency i.e. $\omega = \omega_o + \Delta\omega$ the distributed input impedance is as follows where α is the attenuation constant of the line.

$$Z_{in} = \frac{1}{\frac{\alpha l}{Z_o} + \frac{j\pi}{2\omega_o Z_o}}$$

The input impedance of a lumped parallel RLC resonator near the resonance (see Figure 9) is

$$Z_{in} = \frac{1}{\frac{1}{R_{res}} + 2j\Delta\omega C_{res}}$$

From equating the above, Pozar finds that

$$C_{res} = \frac{\pi}{4\omega_o Z_o},$$

and from this the equivalent lumped inductance is

$$L_{res} = \frac{1}{\omega_o^2 C_{res}}\ .$$

The above quantities for inductance and capacitance are the ones which should be put in $f = 1/2\pi\sqrt{C_{res}L_{res}}$ to find the resonance frequency.

Note that if you use the characteristics impedance formula as $Z_o = \sqrt{\frac{L}{C}}$ and $\omega_o = 2\pi f = 2\pi\frac{1}{4l\sqrt{CL}}$ (recall that L and C are values per unit length), then C_{res} and L_{res} are:

$$C_{res} = \frac{\pi}{4\omega_o Z_o} = \frac{C_{total}}{2} \quad and \quad L_{res} = \frac{1}{\omega_o^2 C_{res}} = \frac{8}{\pi^2} L_{total}$$

This means that the DC equivalent or lumped capacitance of a quarter wavelength resonator is half of its total distributed value [1].

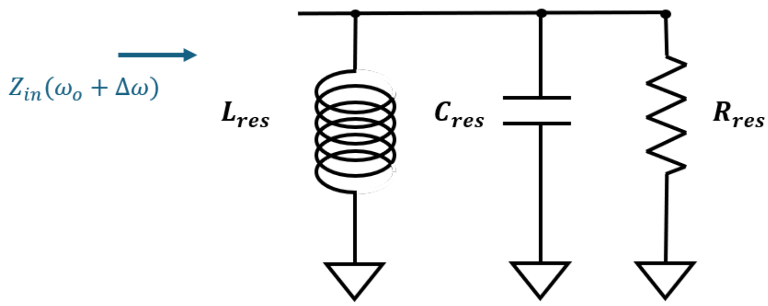


Figure 9. Equivalent circuit of a lumped parallel resonator near the resonance and equating the input impedance with that of a quarter-wavelength resonator.

Conclusion

For further information about optimization, sweep, and statistical analysis refer to the **Quick Start Tutorial for Controlled Impedance Line Designer** (See Appendix A). Users can also design and add new transmission line types to CILD with custom designed ends (short, open), bends, cross sections.

Appendix A

To access the documentation for a specific simulation tool or a method you can always use the **Help** option in the top menu of the main window of Keysight ADS. See Figure 10. After selecting the **Main Window Help** you will be directed to the page shown in Figure 11. Click on **ADS Documentation** on the top left menu and you have a category of tools e.g., **Quantum,** under which there is a list of available documents (see the big green box). Alternatively, you can look at the list on the left side (under ADS logo) and click on **Applications**. After this a new list is opened and you can follow the subdirectories to get to the examples and the relevant documents (subjects) as a html page.

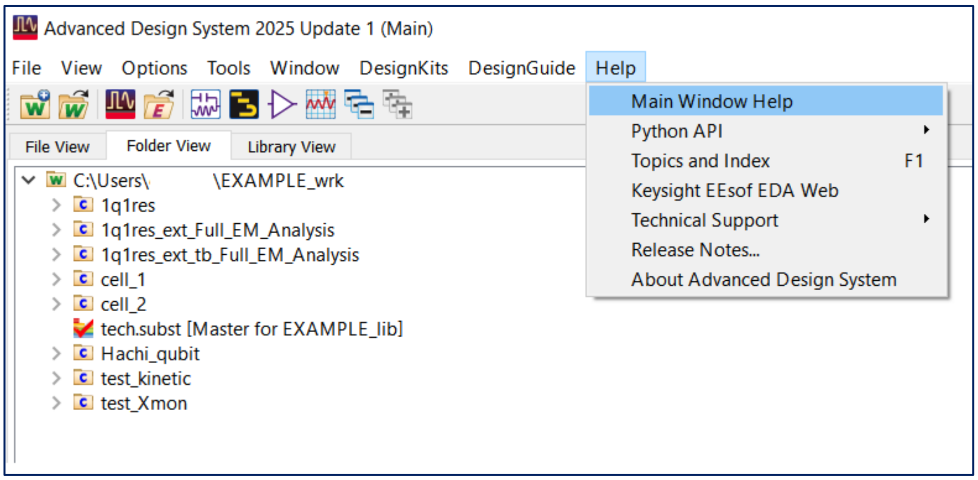


Figure 10. Accessing the documentation using the Help menu in Keysight ADS.

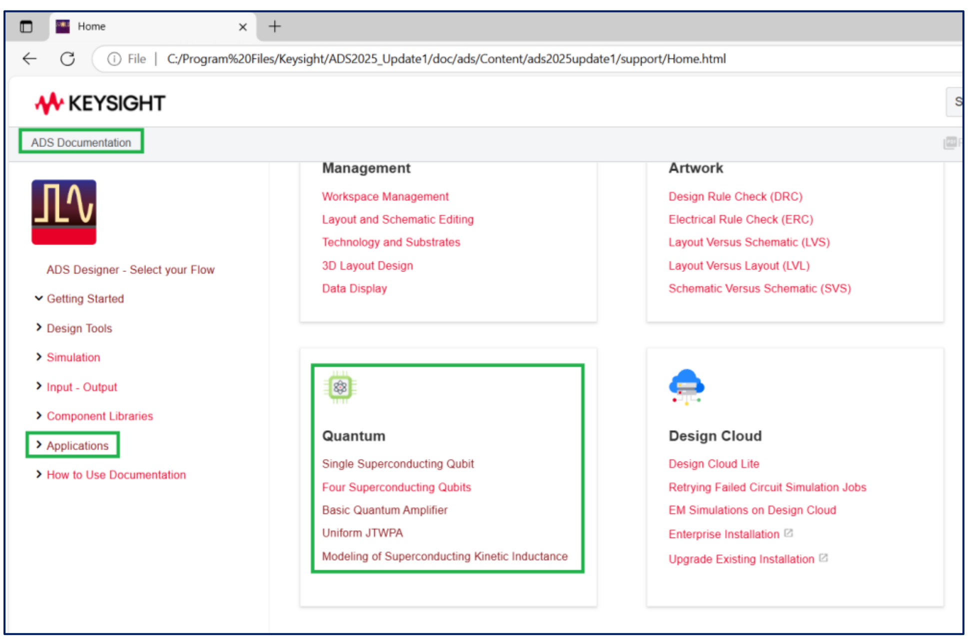


Figure 11. Accessing the help pages in ADS Documentation.

To access the help documentation and examples, you can alternatively open the **Help** option *inside each tool* e.g., when you are working with CILD, you can do this by clicking directly on the **Help** in the top menu. See Figure 12. Then you are directed to a html page with a list for examples and know-hows including **Quick Start Tutorial for Controlled Impedance Line Designer**.

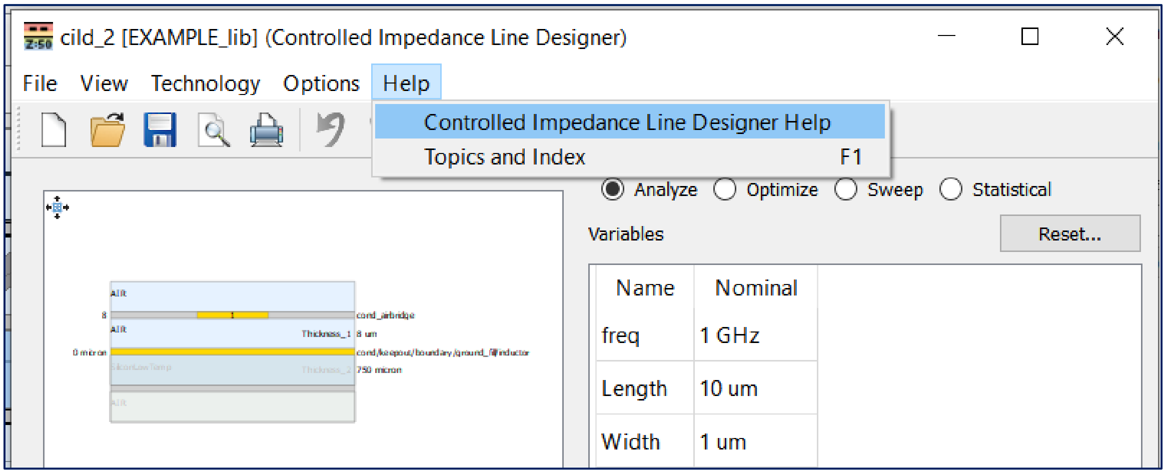


Figure 12. Accessing the documents for CILD within CILD.

References

[1] David M. Pozar, Microwave Engineering, Fourth Edition, John Wiley & Sons Inc., (2011).

[2] Robert E. Collin, Foundations of Microwave Engineering, 2nd Edition, IEEE-Wiley Press, (2000).