APPLICATION NOTE

Cross-Spectrum Noise Measurements

Theory, practice, pitfalls, and countermeasures

KEYSIGHT

Table of Contents

The Cross-Spectral Power Density 4

Generalized Cross Correlation System Model 8

Analysis of Cross-Spectrum 9

Phase Noise Measurement using Cross-Correlation 11

Anti-correlated Noise Sources 11

Collapse of the Cross-Spectral Function 13

Baseband Noise Model 15

Analysis of Correlated Powers 16

Analysis of Uncorrelated Powers 17

Baseband Noise: Simulation vs Measurement 18

Baseband Simulation Setup 21

Simulation Results 26

Phase Noise Cross-spectral Collapse 28

Simulation Setup Using Harmonic Balance 28

Simulation results 33

Anti-Correlated Noise of Wilkinson Power Divider 33

Correlated Noise Cancellation of Termination Resistors 35

Source Resistor Noise Cancellation by WPD Anti-Correlated Noise 36

Total Correlated and Uncorrelated Noise 37

Simulation with Quadrature Phase Errors 39

Reference Phase Noise Cross-Spectral Collapse 42

Measured Cross-Spectral Collapse 43

AM PM Cancellation of Carrier Noise 46

Transient Simulation Results 48

Transient Simulation Setup 48

Anti-Correlated AM with Opposite Polarity Reference Phase Errors 52

Countermeasures to Cross-Spectral Collapse 55
Conclusion 56
Appendix 57
Phase Noise Definition 57
AM Noise Definition 58
Thermal Noise 59
Power Spectral Density 59
Wiener-Khintchine Theorem 60
Transient Baseband Simulations 61
Schematic 61
Only Rs 62
Only Rs and RPD 62
Only R1 and R2 noise 63
All Resistors 63
4-Stage Wilkinson Power Divider 64

The Cross-Spectral Power Density

The “Power Spectral Density” (PSD) of a baseband signal source x(t) is denoted by $S_{xx}(f)$ or $S_x(f)$, and is computed as

$$S_{xx}(f) = |F[x(t)]|^2 = F[x(t)] \cdot F^*[x(t)] \quad (1)$$

where F[x(t)] is the Fast Fourier Transform (FFT) of a sampled x(t), and it may be normalized to a 1Hz resolution bandwidth obtained by dividing the FFT by the tone spacing, which is 1/T, T being the FFT period. The operator “*” is a conjugate function. $S_{xx}(f)$ is always a positive real quantity. From the Wiener-Khintchine Theorem, $S_{xx}(f)$ is the Fourier Transform of the autocorrelation $R_{xx}(\tau)$ of x(t), which is

$$S_{xx}(f) = F[R_{xx}(\tau)] \quad (2)$$

$$R_{xx}(\tau) = E[x(t) \cdot x^*(t-\tau)] \quad (3)$$

The N5511A Phase Noise Test System architecture for measuring the PSD of x(t) is shown in Figure 1. To suppress the system noise contribution of the measurement setup, $S_{xx}(f)$ or $R_{xx}(\tau)$ is measured using a 2-channel cross-correlation system.

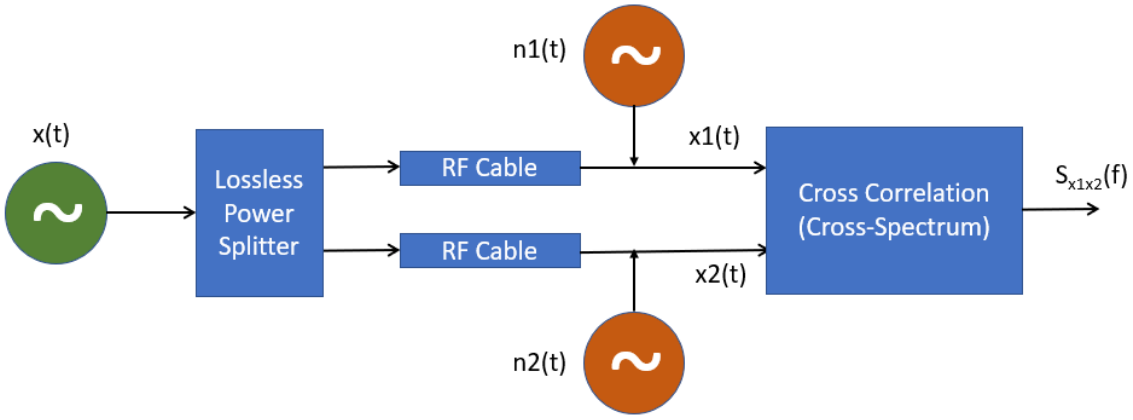


Figure 1. The 2-Channel Cross-Spectrum or Cross Correlation method to measure Sx1x2(f) as an approximation of the PSD Sxx(f) of a baseband signal

The 2-channel cross-correlation system aims to get the autocorrelation. $R_{xx}(\tau)$ of x(t) using the approximation $R_{xx}(\tau) \approx R_{x1,x2}(\tau)$ i.e., the cross correlation between the power splitter signals x1(t) and x2(t) that are derived from the source x(t) and are fed into two separate receiver paths (channels). A Fourier transform of the cross correlation provides the PSD $S_{x1,x2}(f)$ with the expectation that $S_{x1,x2}(f) = S_{xx}(f)$ when the cross correlation is done over a long time. Any noise n1(t) and n2(t) in the two channels, respectively, that are uncorrelated, will get suppressed by the cross correlation $R_{x1,x2}(\tau)$ as described next.

$$x1(t) = x(t) + n1(t) \quad (4)$$

$$x2(t) = x(t) + n2(t) \quad (5)$$

$$R_{x1x2}(\tau) = E[x1(t) \cdot x2^*(t-\tau)] \quad (6)$$

$$R_{x1x2}(\tau) = E[x(t) \cdot x(t-\tau)] + E[x(t) \cdot n1(t-\tau)] + E[x(t) \cdot n2(t-\tau)] + E[n1(t) \cdot n2(t-\tau)] \quad (7)$$

The last 3 terms above tend to zero when computed over a sufficiently long time, as they are the cross-correlation between uncorrelated signals: a) x and n1, b) x and n2, and c) n1 and n2.

Using the Wiener–Khinchin theorem, $S_{x1x2}(f)$ can be computed far more practically in the frequency domain by taking the cross-spectral product as:

$$S_{x1x2}(f) = F[R_{x1x2}(\tau)] = F[x1(t)] \cdot F^*[x2(t)] \quad (8)$$

$$S_{x1x2}(f) = F[x(t)] \cdot F^*[x(t)] + F[x(t)] \cdot F^*[n1(t)] + F[x(t)] \cdot F^*[n2(t)] + F[n1(t)] \cdot F^*[n2(t)] \quad (9)$$

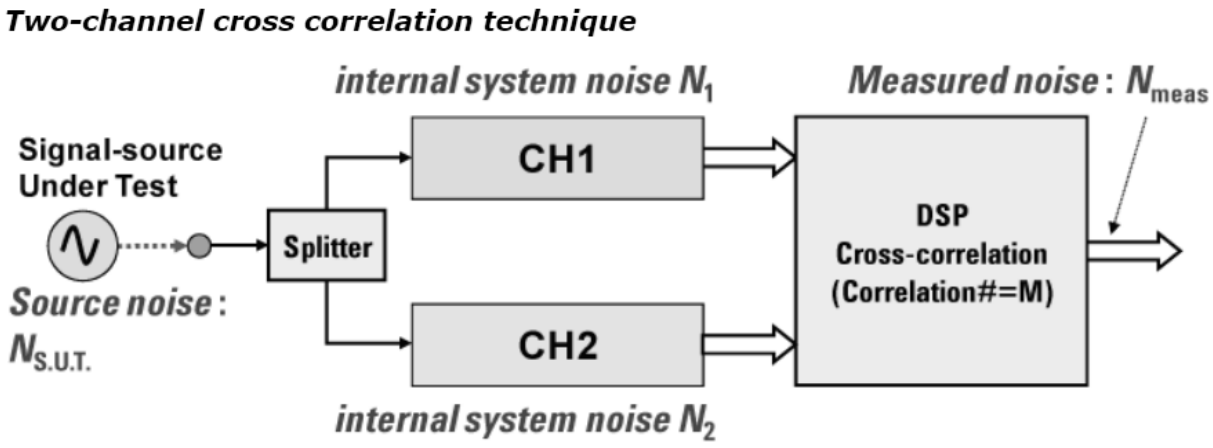
$$S_{x1x2}(f) = X \cdot X^* + X \cdot N1^* + X \cdot N2^* + N1 \cdot N2^* \quad (10)$$

where X = X(f) = FFT[x(t)], and similarly for other terms.

The last 3 terms above are the cross-spectrum of uncorrelated signals, and they will go to zero when FFT is computed over a sufficiently long period of time, or with averaging over multiple FFTs taken over shorter time periods. In practice, when the cross-spectrum is calculated, the contribution of $X \cdot N1^* + X \cdot N2^* + N1 \cdot N2^*$ is reduced by the square root of the number of averages (N) or the observation time. When observation time T or number of averaging N tends to infinity,

$$\boldsymbol{S_{x1x2}(f) = S_{xx}(f)}$$

The 2-Channel Cross Correlation and the suppression of uncorrelated noise is shown conceptually in the figures below.



$$N_{meas} = N_{S.U.T.} + (N_1 + N_2)/\sqrt{M}$$ Assuming N_1 and N_2 are uncorrelated.

M (number of correlation)	10	100	1,000	10,000
Noise reduction on (N_1+ N_2)	-5dB	-10dB	-15dB	-20dB

Figure 2. Noise reduction with the 2-Channel Cross-Spectrum or Cross Correlation method to measure $S_{x1x2}(f)$

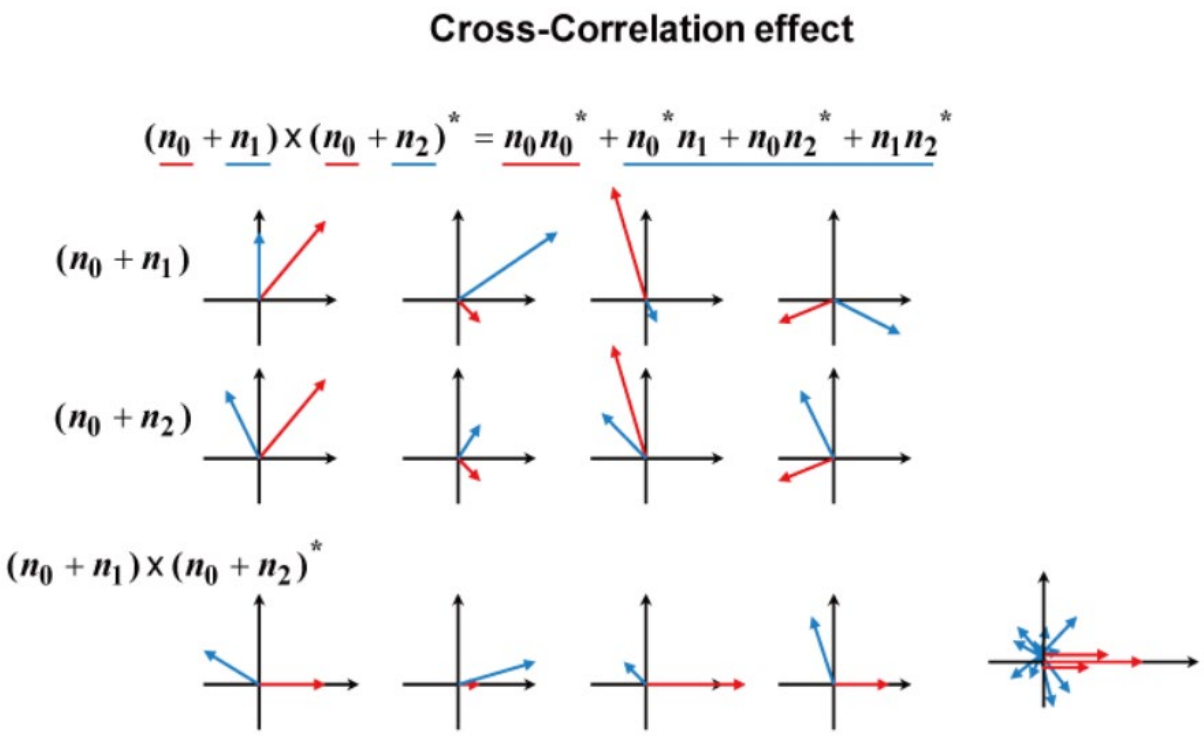


Figure 3. Cross Correlation Noise vectors. n0 is the desired noise to be measured. n0 (top and middle rows) and n0n0* (bottom row) are red arrows. n1 and n2 are the uncorrelated noise in Channel 1 and 2, respectively. They are blue arrows in the top and middle rows. The bottom row shows their power terms in a blue arrow.

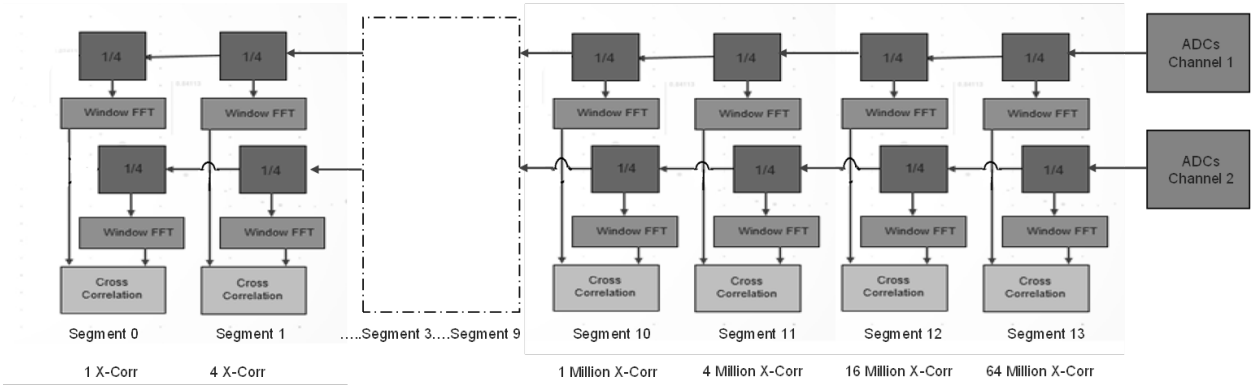


Figure 4. Cross Correlation implementation in FPGA using a cascaded decimation architecture

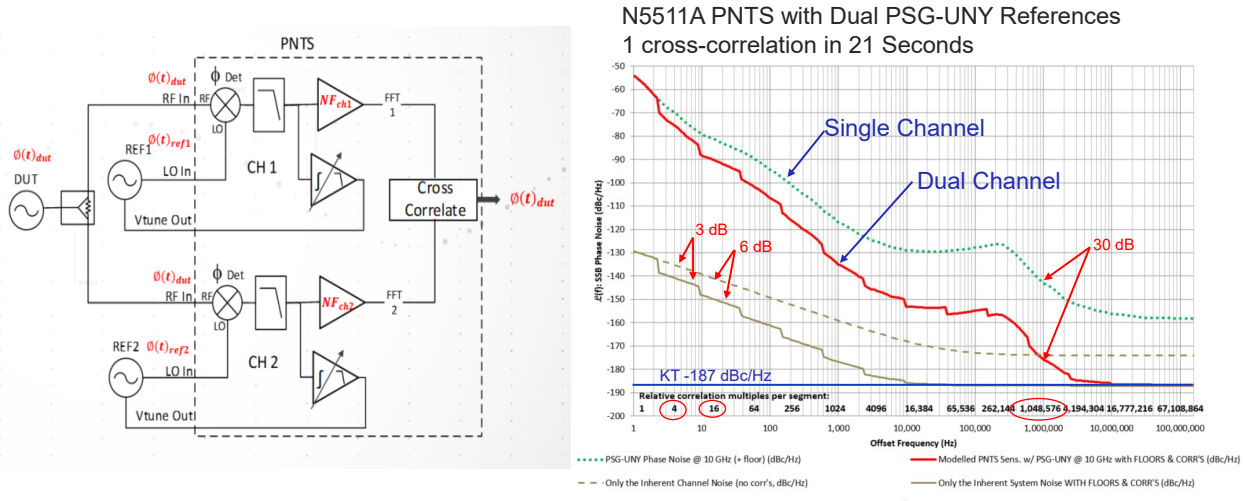


Figure 5. Cross Correlation architecture and Reference Oscillator noise reduction (solid traces). Contribution of N5511A is shown by the solid and dashed brown traces. Reference oscillator noise is shown with the dashed green and solid red traces.

Generalized Cross Correlation System Model

In a real baseband measurement system shown in Figure 6, the source signal x(t) is split by a resistive power splitter (RPD) into equal terms x1(t) and x2(t) that are typically 6 dB less than x(t). For a simplified analysis, we assume the power divider is lossless, but we include a model for the noise of the RPD through noise terms c(t) and d(t) as shown. There are phase shifts in the paths of the signals x1(t) and x2(t) over the two channels. Each channel produces uncorrelated noise n1(t) and n2(t) that leaks from one channel to the other with twice the phase shift.

There is correlated noise common to both channels, and it can be of two types:

Common mode noise (Positively correlated)

It has same polarity for both channels with 0 deg relative phase at low frequencies. c(t) is an example, including source resistor noise. Also, n1(t) in channel 1 and its leakage term in channel 2 have 0 0-degree relative phase at low frequencies. Similarly, n2(t) in channel 2 and its leakage term in channel 1 have 0 0-degree relative phase at low frequencies.

Differential noise (Anti-correlated)

The noise source d(t) is differential, with opposite polarities in each channel or 180-degree relative phase.

At higher frequencies, the correlated noise terms could also have a relative phase that is different from either 0 or 180 degrees for the common mode and differential noise, respectively, and depends on each channel's phase shift.

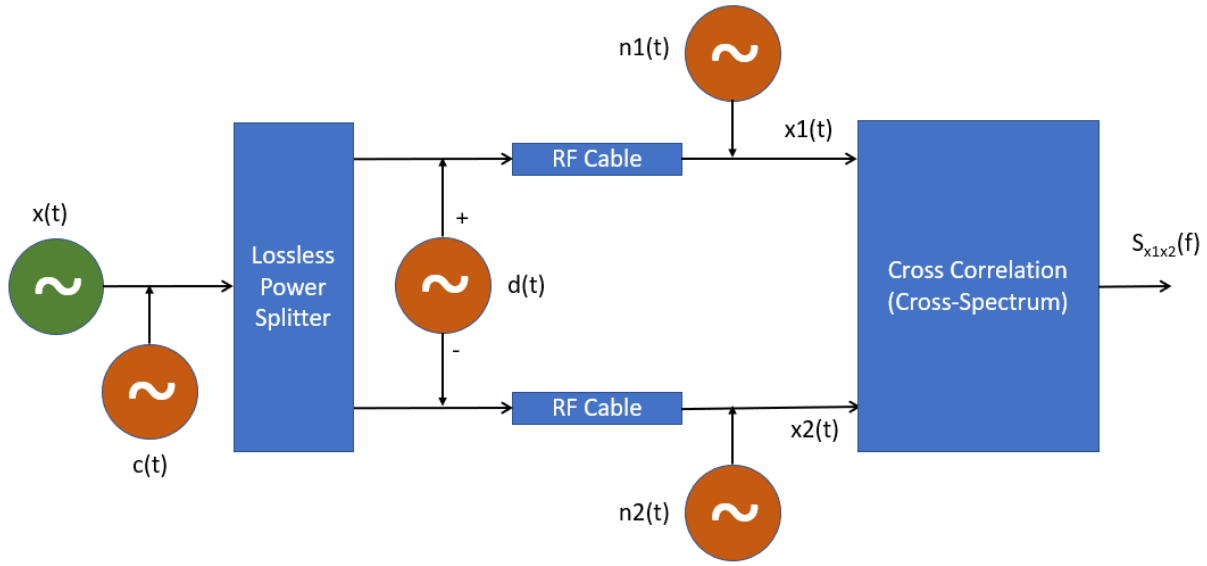


Figure 6. Baseband Noise measurement using 2-channel cross-correlation (or cross-spectrum)

Analysis of cross-spectrum

$$x1(t) = x(t) + c(t) + d(t) + n1(t) + kn2(t) \quad (11)$$

$$x2(t) = x(t) + c(t) - d(t) + kn1(t) + n2(t) \quad (12)$$

$x(t)$ is the wanted signal (or noise) to be measured, $c(t)$ is a common mode noise, $d(t)$ is differential noise, $n1(t)$ is Channel-1 noise with some leakage to Channel-2 as $kn1(t)$, $n2(t)$ is Channel-2 noise with some leakage to Channel-1 as $kn2(t)$. All these signals may have some phase shift that is not explicitly shown in the time domain but will be included in the frequency domain representation below.

$$X1 = Xe^{j\theta x1} + Ce^{j\theta c1} + De^{j\theta d1} + N1 + kN2e^{j\theta n} \quad (13)$$

$$X2 = Xe^{j\theta x2} + Ce^{j\theta c2} - De^{j\theta d2} + kN1e^{j\theta n} + N2 \quad (14)$$

The θ terms are frequency-dependent and are phase shifts due to RF cables and ADC relative clock skew between Channels 1 and 2 digitizers. They are NOT the random phase of the noise voltages. The random phases of the noise sources are in the complex quantities X, C, D, N1, and N2. θn is the deterministic phase shift of noise leaking from one channel to another.

The cross-spectrum is

$$S_{x1x2}(f) = X1 \cdot X2^* \quad (15)$$

$$\boldsymbol{S_{x1x2}(f) = XX^*e^{j\Delta\theta x} + CC^*e^{j\Delta\theta c} - DD^*e^{j\Delta\theta d} + kN1N1^*e^{-j\Delta\theta n} + kN2N2^*e^{+j\Delta\theta n} + U} \quad \textbf{(16)}$$

The first 5 terms of Eq. 16 are correlated. U is the sum of the complex cross-spectrum between uncorrelated noise and is expanded below in Eq. 17. Its expected value is reduced by the square root of the number of averages (N) or the observation time. The expected value of the 5 correlated terms is not affected by averaging.

$$U = Xe^{j\theta x1}[C^{j\theta c2} - De^{j\theta d2} + kN1e^{j\theta n} + N2]^* + Ce^{j\theta c1}[Xe^{j\theta x2} - De^{j\theta d2} + kN1e^{j\theta n} + N2]^* + \cdots \quad (17)$$

The 5 correlated terms of Eq. 16 are shown below in Figure 7 with small phase angles at lower frequencies. DD* has a negative sign and is anti-correlated to the other terms that have positive signs. The kN1N1* and kN2N2* have conjugate phases, and their imaginary components cancel each other and are anti-correlated to each other on the imaginary axis.

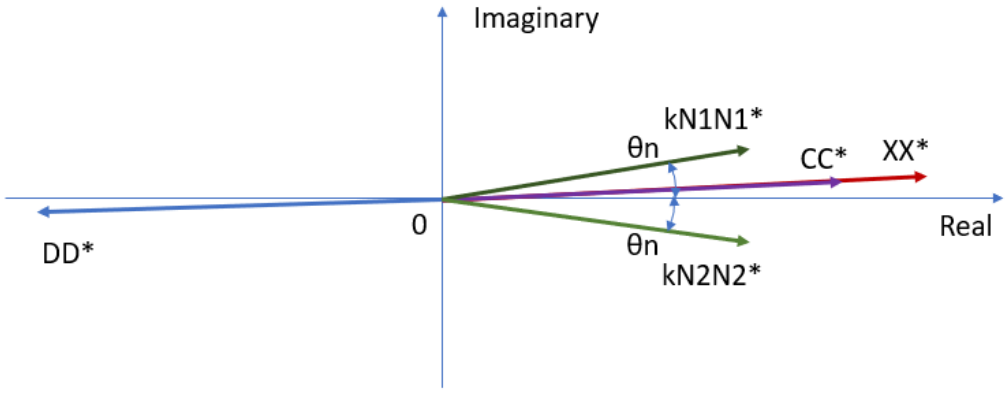


Figure 7. Correlated vector terms and their phase shifts at lower frequencies. DD* is anti-correlated, while others are positively correlated. kN1N1* and kN2N2* have opposite phases and can annihilate each other when the phase magnitudes become 90 deg. The DD* term can cancel out the XX* source resistance noise completely in some situations, as shown later.

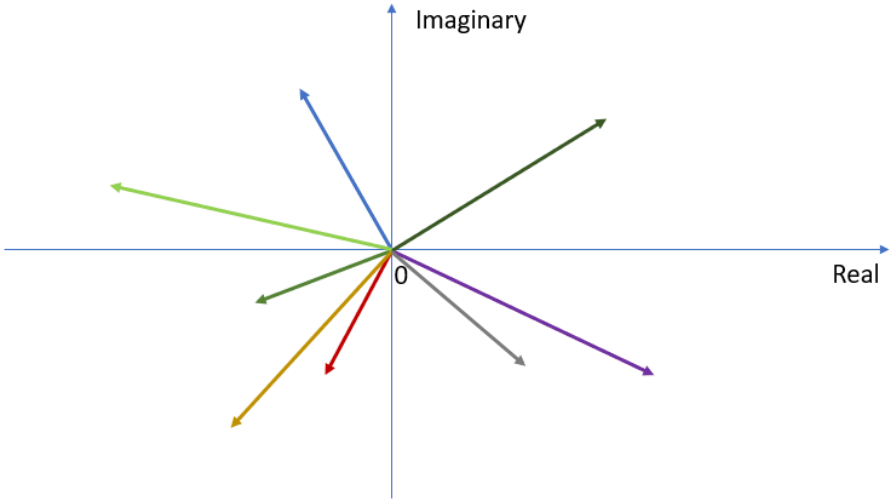


Figure 8. Uncorrelated noise terms have random phases arising from pair-wise uncorrelated noise voltages

Phase noise measurement using cross-correlation

A 2-channel Cross-Correlation phase noise measurement system is depicted in Figure 9. The power splitter is typically the Wilkinson Power Divider (WPD), which has improved isolation between the two channels. M1 and M2 are phase detectors (down converters) with reference local oscillators Ref1 and Ref2 locked to the source signal x(t) in quadrature. To keep the analysis like the baseband model, we consider x(t) to be the phase noise and AM noise of the source signal, and x1(t) and x2(t) are its lossless copies through the WPD. The uncorrelated noise in each channel is shown as n1(t) and n2(t) before the phase detectors M1 and M2. These noise terms leak across to the other channel through the WPD with typically 20 dB attenuation. Baseband noise added after the phase detectors M1 and M2 have higher channel-channel isolation, as they must traverse through the down converters. They will mostly add to the uncorrelated noise that will be suppressed by the correlation gain.

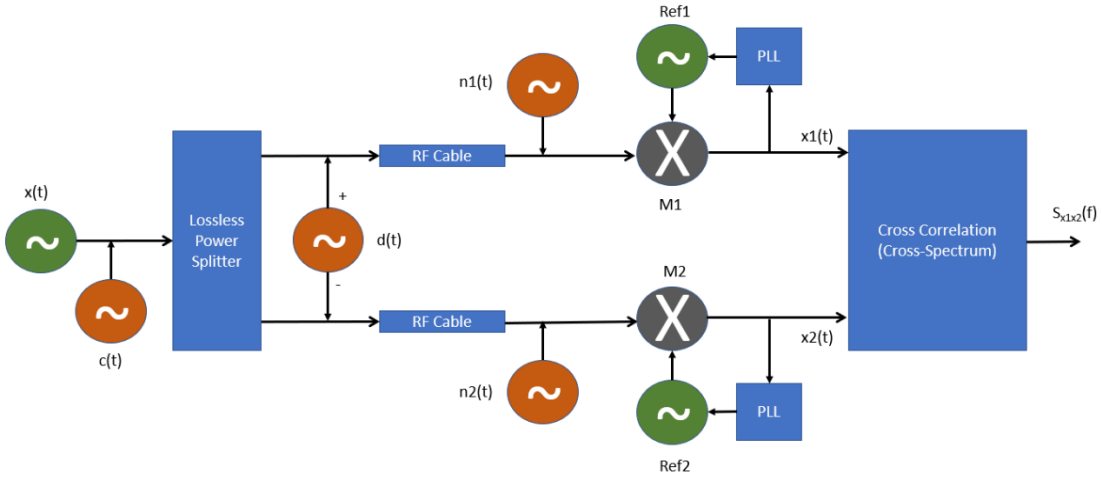


Figure 9. Phase Noise cross-correlation measurement at RF and microwave frequencies using Phase Detectors with reference signals Ref1 and Ref2 that are in quadrature with the source signal.

Anti-correlated noise sources

There are three sources of anti-correlated noise.

1. **WPD shunt resistors**

The anti-correlated noise d(t) is due to the shunt resistor at the WPD output.

2. **Noise leaking from one channel to the other**

Noise n1(t) and n2(t) leak from one channel to the other, creating two autocorrelation noise power terms whose phases are zero at low frequencies. At higher frequencies, typically around 10 MHz and odd harmonics of it, all the way to tens of GHz (upper limit of measurement setup frequency range of carrier), their relative phases are 180 deg and their autocorrelation noise power terms annihilate each other. This also occurs for the reference phase noise from Ref1 and Ref2 due to leakage of the LO to the RF port of the down converters M1 and M2, and subsequent leakage to the other channel through the WPD. If the phase shift of the LO leakage within the down converters is ignored, then the sum of the autocorrelation noise powers from phase noise has the same frequency response as that from n1(t) and n2(t).

3. **AM noise in the source x(t)**

This AM noise can produce anti-correlated noise if the phase detectors M1 and M2 are not operating in the same phase quadrant, i.e., if one channel has a relative phase between LO and RF signals of less than 90 degrees while the other channel has greater than 90 degrees. This implies that the quadrature phase errors are of opposite signs. The AM and PM conversion gain and the phase of the AM conversion gain of the phase detectors are shown in Figure 10. The phase of the AM gain is positive for the RF-LO relative phase less than 90 degrees, and negative for greater than 90 degrees. If the quadrature phase errors are of opposite signs in channels 1 and 2, then the source AM noise will be of opposite signs in each channel, showing up as anti-correlated noise similar to d(t). If the quadrature phase errors are of the same sign in channels 1 and 2, then the source AM noise will be of the same sign in each channel, showing up as correlated noise. If the source AM noise is equal to the phase noise after the down conversion at some offset frequency, they could cancel each other if the quadrature phase errors are of opposite signs, i.e., if the AM noise is anti-correlated. Note that the phase detectors typically have 20 dB AM suppression, so the AM noise needs to be 20 dB higher than the phase noise for this cancellation to occur.

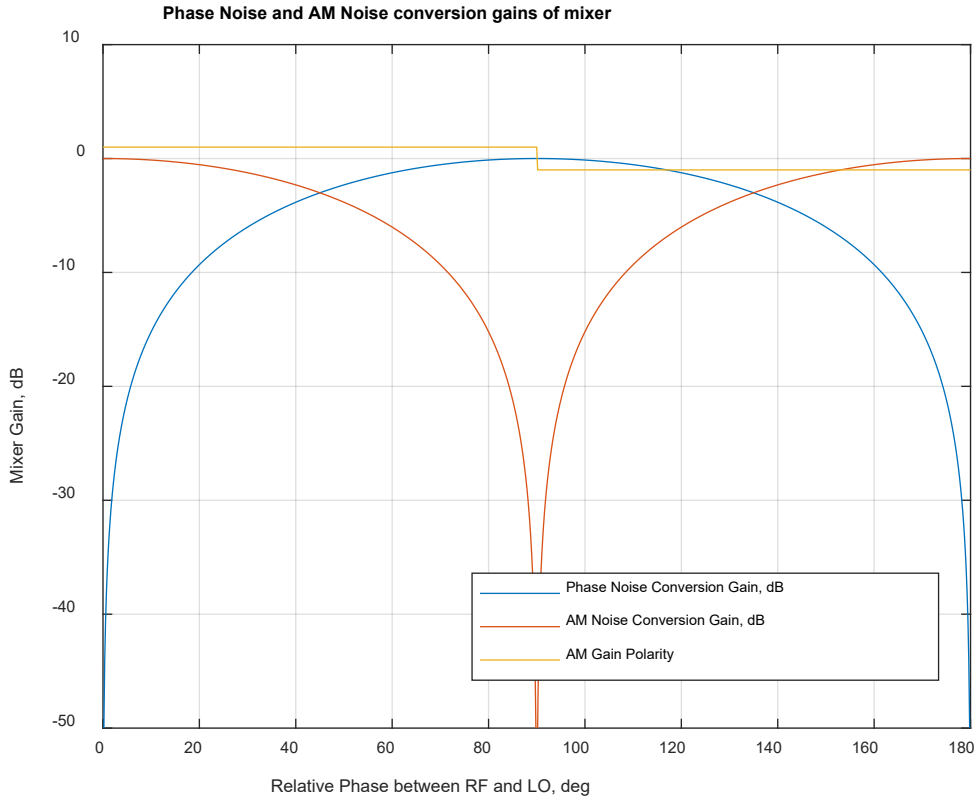


Figure 10. Mixer Conversion Gain and Polarity for Phase Noise and AM Noise. AM Gain polarity is positive below 90 degrees and negative above 90 degrees. The Phase Noise Gain is always positive. AM Gain is cos(θ) and PM Gain is sin(θ) for noise voltage.

Collapse of the Cross-Spectral Function

Each cross-spectrum term XX*, CC*, -DD*, kN1N1*, and kN2N2* in Eq. 16 is a positive or negative real power quantity and therefore correlated or anti-correlated, even though their corresponding voltage sources X, C, D, N1, and N2 are totally uncorrelated. These real cross-spectrum terms are phase rotated to complex quantities when multiplied by the exponential terms in Eq. 16 that are due to

a) the relative phase difference between the two channels for the source noise XX*, CC*, and power divider noise DD*

b) the total channel 1 and channel 2 phase shift for the termination resistor noise kN1N1* and kN2N2* (and reference oscillators phase noise for phase noise measurements).

The phase shifts are not random but constant over time and are only dependent on circuit paths in the channels. These complex terms are correlated and add vectorially. At low frequencies, the phase rotation by the exponential multipliers is negligible, but it can become significant at higher frequencies. Unlike a normal PSD, the cross-PSD is a complex quantity.

- The cross-power spectral density computation is unable to discern between these correlated signals, resulting in the combination of all of them and thereby creating spectral collapse artifacts. A common occurrence is when amplitude modulation (AM) noise CC* from a source contaminates a phase modulation (PM) XX* measurement. The AM noise CC* becomes anti-correlated like -DD* when the mixers M1 and M2 as shown in Figure 6 don't operate in the same quadrant i.e., in one channel the Reference Oscillator Ref1 is just below 90 deg and in the other channel Ref2 is slightly above 90 deg relative to the RF signal phase, or vice versa. The AM conversion gain in such a case will have opposite polarities for the two channels, as shown in Figure 10. This is covered in more detail under Phase Noise measurement.
- The DD* term is anti-correlated as it is subtracted and can annihilate the noise from other terms, leading to spectral collapse. Its source is typically the differentially connected resistor in the Wilkinson Power Dividers (WPD) and the Delta-topology Resistive Power Divider (RPD).
- At low frequencies, the two noise terms kN1N1* and kN2N2* add in phase, but at some higher frequency, like approximately 10 MHz, when the leakage noise-path phase shift $\Delta\theta n$ is 90 degrees, the two noise terms cancel as they both become Imaginary with opposite signs, i.e., their phase difference is 180 deg. This can lead to a large collapse of the PSD, especially if the XX* - DD* is already net zero in some cases involving baseband noise measurement using a resistive power divider (RPD), as shown later. At odd multiples of the first null frequency, the spectral collapse repeats because the relative phase difference will be odd multiples of 180 degrees. This phenomenon is observed not only for baseband noise measurements, but also for Phase Noise at RF and Microwave frequencies.

What Eq. 16 tells us is that at any frequency *f* where the average magnitude of signal *X(f)* is equal to that of signal *D(f)*, the magnitude of the combined cross-spectrum collapses to zero. Any contribution of the desired signal *x(t)*, or the interferer *d(t)*, to the cross-spectral density is eliminated. This occurs even though signals *x(t)* and *d(t)* are completely uncorrelated. If the PSDs of the two signals are exactly equal, the amount of observed cancellation is limited to $\sqrt{N}$, where *N* is the number of averages. If *X(f)* and *D(f)* have the same shape or slope versus frequency, entire octaves or decades of spectrum can be suppressed and be grossly under-reported. If the PSDs XX* and DD* are not exactly equal, a partial cancellation still occurs.

In the N5511A phase noise test setup for cross correlation, the cross-spectral collapse is produced by the shunt resistors in both the Wilkinson Power Divider (WPD) and the Resistive Power Divider (RPD) that are used to split the DUT signal output for feeding the two channels. The RPD has just one equivalent shunt resistor (either Delta or Y configuration) while the WPD has typically one or more shunt resistors between the Channel 1 and 2 signal paths, and these resistors produce anti-correlated noise “-DD(f)” that leads to the cross-spectral collapse.

Interestingly, due to excessive phase shifts from the total leakage path lengths of combined Channel 1 and Channel 2 for the port terminating resistors, noise n1(t) and n2(t) leaking from one channel to the other, the autocorrelated noises of these terminating resistor can cancel each other, creating spectral collapse or dips at higher frequencies. This occurs not just for baseband noise measurements but also for phase noise measurements, where the Channel 1 and Channel 2 reference oscillators' phase noise contribute to n1(t) and n2(t).

The next section compares a simulated model of the cross-spectral collapse to measurements.

Baseband Noise Model

The cross-correlation is done between Channels 1 and 2 using the equation:

Power(f) = v1(f)*conj(v2(f))

Both v1(f) and v2(f) are the vector sum of noise voltages (terms) from all the noise sources. In our analysis, we will consider the cross-correlation between each two noise terms, one taken from v1 and the other from v2, that results in a noise power term. Note that cross correlation is defined in the time domain, which is equivalent to multiplication in the frequency domain with the second multiplicand conjugated.

The cross-correlation produces 2 types of noise power terms:

1. **Correlated power terms**

This is produced when both Channel 1 and 2 signal terms are from the same source. Even though it is a cross-correlation between the two channels, because the noise signals originate from the same source, mathematically, it is an autocorrelation. Common-mode signals produce a positive real power term. An example is noise from Rs in the Source and noise from termination resistors R1 and R2.

Differential signals produce a negative real power term. An example is Rdiff in the resistor power divider.

- The autocorrelation terms do not change the mean (expected) power value when averaged over multiple correlations.

For example, the noise from resistor R1 feeds Channel 1 by direct connection and leakage through the RPD to Channel 2 with 6 dB loss and some phase shift through the TLs. Or noise from Rs feeds both channels with some relative skew between the signal paths, including any ADC clock skew.

- At higher frequencies, the autocorrelation power terms can become complex due to asymmetric phase shifts in the paths of each channel from the source resistor.
- When phase shifts occur in opposite directions, the autocorrelation powers from different noise sources could subtract, leading to a dip in the total power.

An example is the autocorrelation powers of R1 and R2 subtracting at 8 MHz and harmonics due to the phase shifts in the RF Cable path length between channel 1 and 2 ports through the RPD. At low frequencies, the autocorrelation powers of each R1 and R2 are positive real.

- At some frequency, their relative phase will be 180 degrees, and their autocorrelation powers will annihilate, creating a dip in the noise spectrum.

Longer the RF cables, lower the dip frequency. Dips also occur at odd harmonics of the first dip.

2. **Uncorrelated power terms**

This is produced when Channel 1 and 2 signal terms in the cross correlation are from the different (uncorrelated) noise sources. For example, the noise from resistor R1 feeding Channel 1 by direct connection, and R2 feeding Channel 2 also by direct connection. Or source resistance Rs feeding Channel 1, and R1 feeding Channel 2 through the leakage path.

Analysis of correlated powers

Each resistor in the circuit is a noise source. The FFT or frequency domain representation of any one of these voltages in a 1Hz bandwidth is

$V(f) = M*\exp(j\theta(f)))$

where M = 4kTR is the magnitude and θ(f) the random phase at the source voltage at frequency f. Here k=Boltzman constant, T= temperature in Kelvin (300 deg), R= resistance (50 ohms in our case).

The above noise voltage, when observed at the Channel 1 and Channel 2 ports, is given by

$V1(f) = M1(f)*\exp(j[\theta(f) + 0.5*\phi1(f)])$

$V2(f) = M2(f)*\exp(j[\theta(f) - 0.5*\phi2(f)])$

where $\phi1(f)$ is the phase skew between the noise source and channel 1, and $-\phi2(f)$ between the noise source and channel 2. These are measured from the position of the noise source in the circuit to the channel ports. A1(f) and A2(f) are the amplitudes at each channel, and they may not be equal due to unequal attenuation from the noise source voltage to the channel ports.

The autocorrelation noise power is

$p(f) = V1(f)*conj[V2(f)] = [M1(f)*M2(f)]*\exp[j\phi(f)]$, or

$p(f) = Mac(f)*\exp[j\phi(f)]$

where

$\phi(f) = \phi1(f) + \phi2(f)$ is the phase skew

$Mac(f) = M1(f)*M2(f)$ is the magnitude of the autocorrelation (magnitude of power)

p(f) is independent of the phase θ(f) of the original signal and is only dependent on the phase skew ϕ(f) and amplitude skew of the two channels for the given noise source.

The phase skew ϕ(f) includes the RF Cable length mismatches and ADC clock delay skew between the two channels. Even with equal cable lengths, there will be skew for R1 and R2 terminating resistors of each channel as they are placed closer to one channel than the other. However, for the Source resistance Rs, the skew will depend on the path length mismatch between the two channels. ADC clock skew will be common to all noise sources. In the simulations shown next, the path delay mismatch between the transmission lines was set to 90 degrees at 500 MHz, which is close to the sampling frequency in the N5511A. The path length was set to 2700 degrees at 500 MHz, leading to the first dip frequency at 8 MHz.

Analysis of uncorrelated powers

Noise voltages from sources A and B at Channels 1 and 2 are

VA1(f) = MA1(f)*exp(j[θA(f) + 0.5*ϕA1(f)]) from Source A at Channel 1 port

VA2(f) = MA2(f)*exp(j[θA(f) - 0.5*ΦA2(f)]) from Source A at Channel 2 port

VB1(f) = MB1(f)*exp(j[θB(f) + 0.5*ϕB1(f)]) from Source B at Channel 1 port

VB2(f) = MB2(f)*exp(j[θB(f) - 0.5*ϕB2(f)]) from Source B at Channel 2 port

where ϕB1(f), ϕB2(f), etc. are the phase skews between channel 1 and channel 2, which are measured from the position of the noise sources A and B in the circuit to the channel ports 1 and 2. Θ1(f) and θ2(f) are random phases of the noise.

The two cross-correlation noise power terms are

Term 1: A at Channel 1, B at Channel 2

PAB(f) = VA1(f)*conj[VB2(f)] = [MA1(f)*MB2(f)]*exp(j[θA(f)-θB(f) + 0.5ϕA1(f) + 0.5*ϕB2(f)])

Term 2: B at Channel 1, A at Channel 2

PBA(f) = VB1(f)*conj[VA2(f)] = [MB1(f)*MA2(f)]*exp(-j[θA(f)-θB(f) - 0.5ϕA2(f) - 0.5*ϕB1(f)])

The skew ϕA1(f), ϕA2(f), etc, are constant over time and can be ignored at low frequencies, at which the phase of PAB and PBA are equal and opposite, even though they are random from one correlation to another. When the two powers are combined vectorially for each correlation, and if their magnitudes are equal, they produce only a real term whose average will tend to zero with averaging over multiple cross-correlations. At higher frequencies, when the skew increases, it will produce an imaginary term that will also tend to zero with averaging.

Baseband Noise: Simulation vs Measurement

The two-channel cross-correlation method for baseband noise measurement and simulation is shown in Figure 11a. The Source power is split by a Resistive Power Divider (RPD) into two channels. The voltage signals v1 and v2 at the termination ports of these two channels are cross-correlated in the box shown using a set of equations to measure the Source noise while suppressing other uncorrelated noise terms by the correlation gain. The simulated noise results in dBV/Hz are in Figure 11b with some traces offset slightly to avoid overlap. The noise terms shown are:

- pTotal = total power
- pCorr = total correlated power as a sum of all the 2-channel autocorrelation of each noise source. Note that each noise source produces correlated power when its autocorrelation is done between the two channels.
- pUncorr = total uncorrelated power as a sum of all the 2-channel cross correlation between every pair of uncorrelated noise sources, divided by correlation gain (noise suppression). Note that when cross-correlation is done between uncorrelated sources, the result is suppressed by the correlation gain of 5*log10(N), where N is the number of cross-correlations.
- p0 = 50 ohms thermal noise floor of -187 dBV/Hz at 300 deg K.

A dip in total power occurs at 8 MHz when the RF Cable length is set to 2700 deg at 500 MHz in the simulation. This corresponds to the actual measurement on the N5511A using very short RF cables that don't add much to the phase shift. Most of the phase shift is internal to the N5511A.

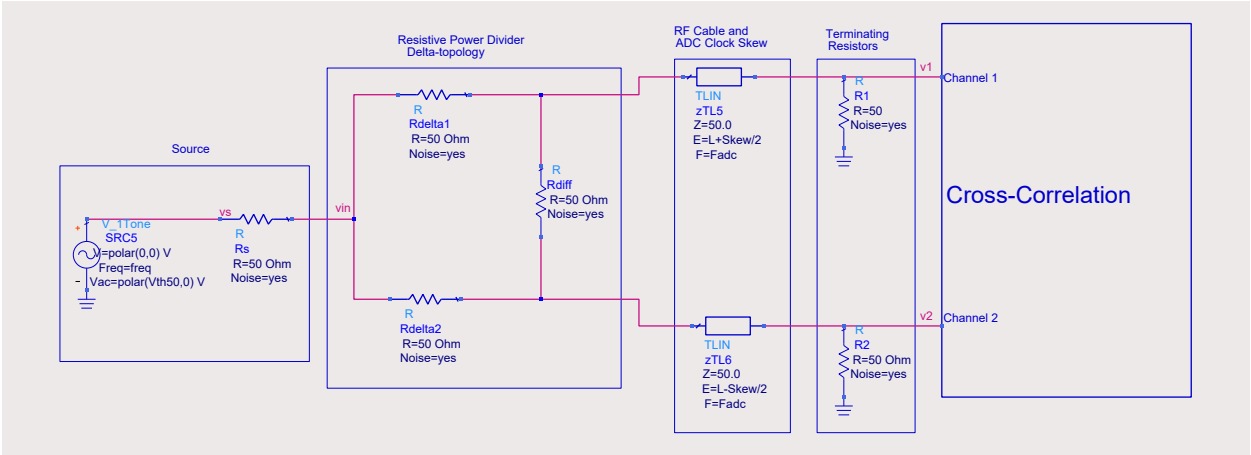


Figure 11a. Cross-Correlation Model for measuring Baseband Noise from Source

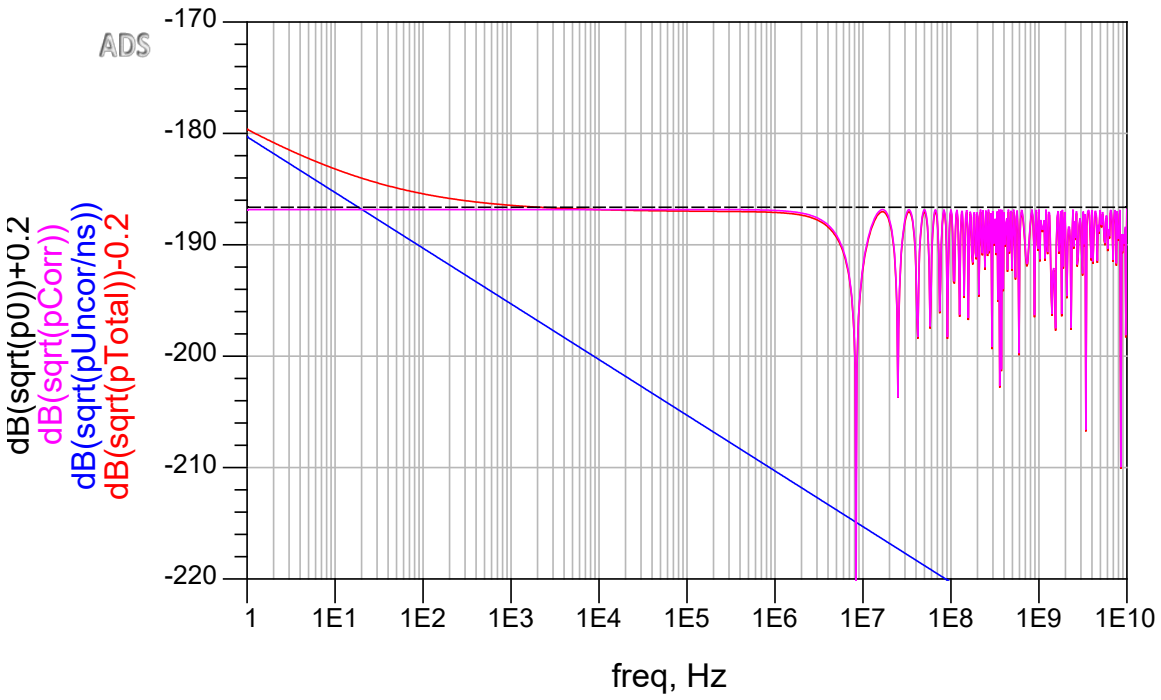


Figure 11b. Various noise components in dBV/Hz over frequency.

Measurement with the N5511A is shown in Figure 11c. The simulated total power pTotal of Figure 11b should be compared to the measurement of Figure 11c. The theory behind the results are discussed in the next section.

- pTotal = total power
- pCorr = total correlated power as a sum of all the 2-channel autocorrelation of each individual noise source.
- pUncorr = total uncorrelated power from 2-channel cross correlation between every pair of (uncorrelated) noise sources, divided by correlation gain (noise suppression)
- p0 = 50 ohms thermal noise floor of -187 dBV/Hz at 300 deg K.

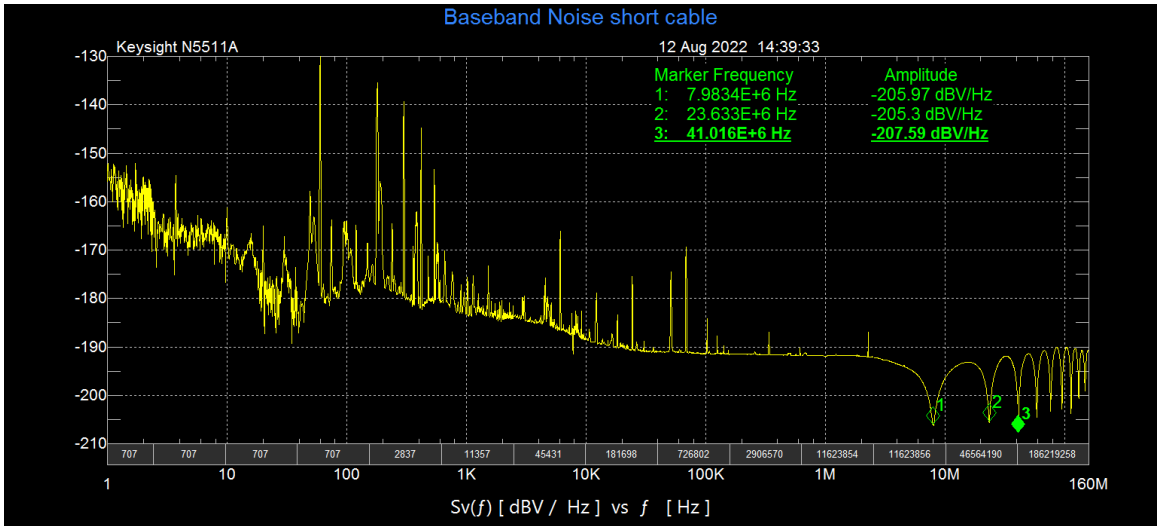


Figure 11c. Measured noise floor with N5511A using 2-channel cross correlation

Impact of channel delay

In the baseband noise floor measurements shown in Figure 11d for the N5511A using a Resistor Power Divider (RPD), we can clearly see the first dip in power at 8MHz for very short RF cables and 5.9 MHz for a long RF cable. The dips occur at odd harmonics of the first dip frequency.

The uncorrelated noise from the two channels is suppressed by 5*log(N) where N is shown at the bottom axis just above the frequency values. The uncorrelated noise mostly shows up below 10 kHz where the noise floor exceeds the thermal -187 dBV/Hz. This trend is like that of the simulated results that did not include the uncorrelated internal noise of the N5511A.

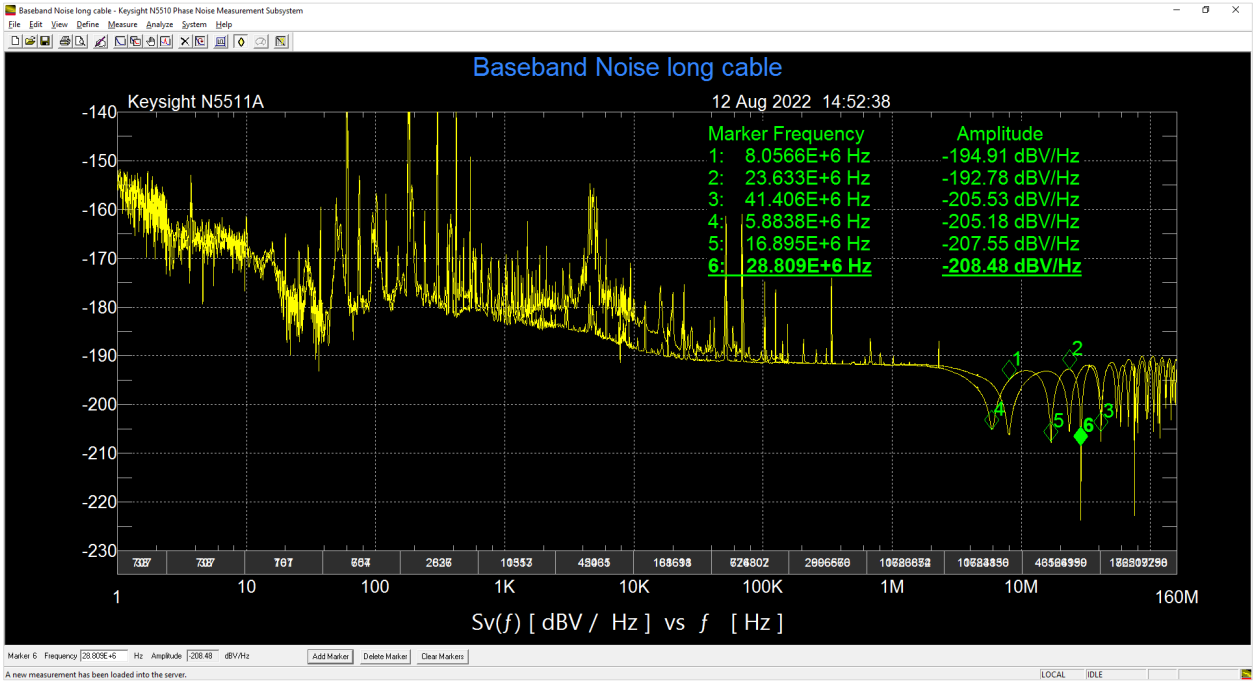


Figure 11d. 50 ohms terminated Baseband Noise floor in “dBV/Hz” units. The Noise Floor is at the Thermal Noise of -187 dBV/Hz at around 10 kHz and higher frequencies.

Baseband simulation setup

A Resistive Power Divider (RPD) is used for the baseband noise measurement from the source resistance Rs, as shown in Figure 12a for the 2-channel cross-correlation method. There are in total 6 resistors as shown. The noise from each resistor is measured in a separate but identical circuit as shown in Figure 12b. The autocorrelation noise power equations are in Figure 12c. The phase ϕ(f) for each autocorrelation noise power is shown in Figure 12d. The Magnitude, Real, and Imaginary components of the autocorrelation noise power are in Figure 12e.

The equations for the Uncorrelated Noise Powers are in Fig 2f and the simulated values are in Fig 2g.

System calibration

All the cross-correlation power values are as-is, i.e., for Rs, it has the 6 dB loss from the RPD. Only the reference thermal noise p0 at -187 dBV/Hz (dashed black trace) is without any attenuation, as it is simulated separately, as shown in Figure 12a for the node v0. As the true value of noise density of Rs is -187 dBV/Hz, while the simulated autocorrelation power shows -193 dBV/Hz in Figure 12e due to the RPD loss, we should add 6 dB to all the power traces except p0.

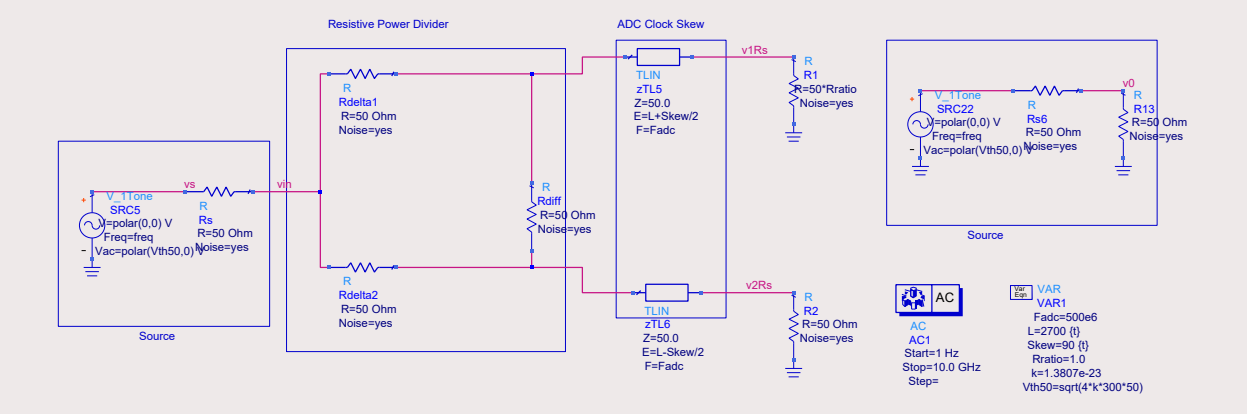


Figure 12a. 2-channel cross-correlation model for baseband noise from Rs

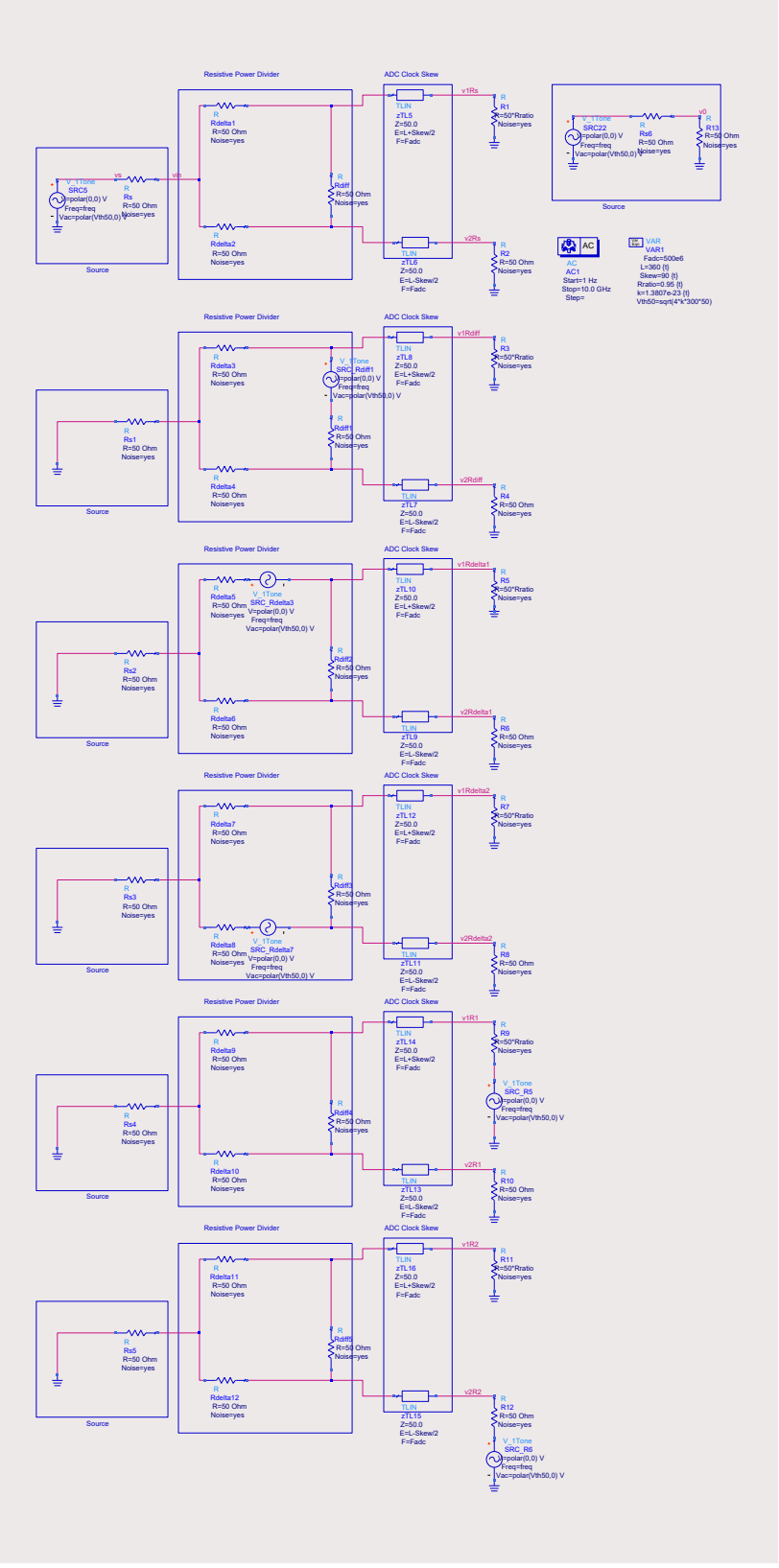


Figure 12b. 2-channel cross-correlation model for baseband noise with a separate but identical circuit for each noise contributor Rs, Rdiff, Rdelta1, Rdelta2, R1, and R2. (ADS Design: cell_AC_multiple)

```
Eqn=v0*conj(v0)
Eqn=v1Rs*conj(v2Rs)
Eqn=v1Rdiff*conj(v2Rdiff)
Eqn=v1Rdelta1*conj(v2Rdelta1)
Eqn=v1Rdelta2*conj(v2Rdelta2)
Eqn=v1R1*conj(v2R1)
Eqn=v1R2*conj(v2R2)

Eqn=pRs+pRdiff+pRdelta1+pRdelta2+pR1+pR2

Eqn=abs(pCorr)+abs(pUncor/ns)
```

Figure 12c. Equations for correlated and anti-correlated noise terms. 2-channel complex cross-correlation power computation for each individual noise contributor.

- pRs is the cross-correlation power of source resistor Rs
- pRdiff is the cross-correlation power of the shunt resistor Rdiff of the RPD
- pRdelta1 and pRdelta2 are cross-correlation power of Rdelta1 and Rdelta2 of the RPD
- pR1 and pR1 are cross-correlation power of R1 and R2
- pTotal is the vector sum of all the complex cross-correlation powers
- Note: 50 ohms thermal floor reference p0 of -1 87 dBV/Hz is simulated in a separate circuit as shown for voltage v0.
    - p0 is the autocorrelation power of 50 ohms thermal noise voltage

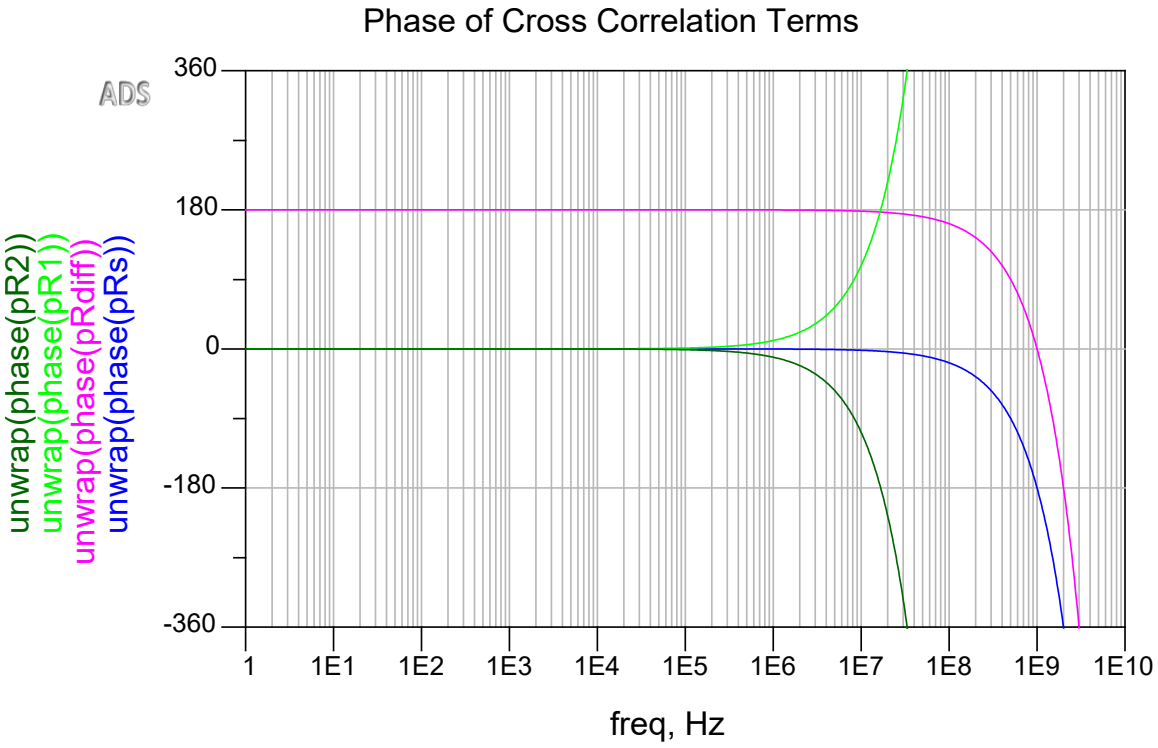


Figure 12d. Phase of 2-channel cross-correlation power of Rs, Rdiff, R1, and R2 noise sources.

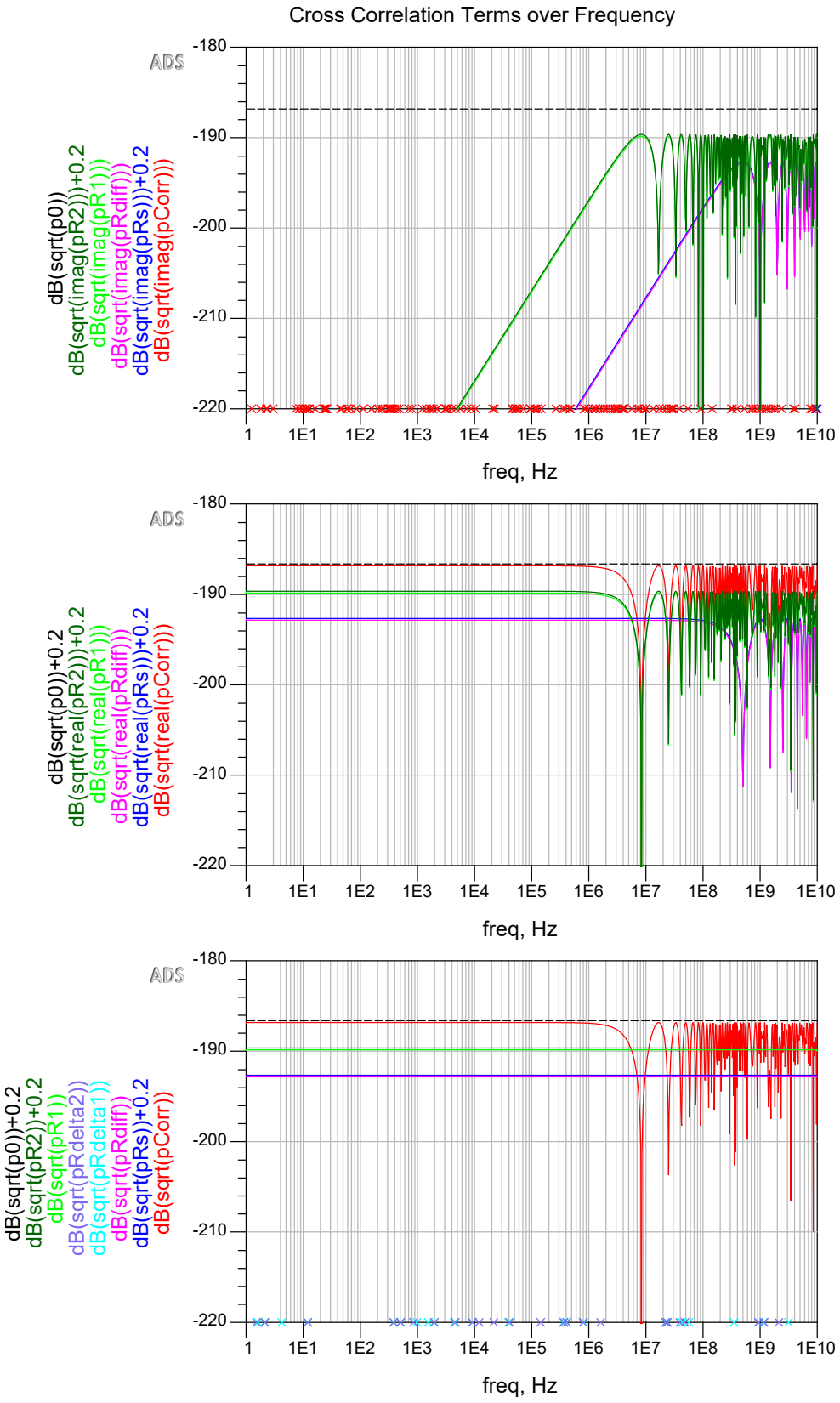


Figure 12e. Magnitude, real and imaginary components of autocorrelated powers in dBV/Hz

```
Eqn sR1=v1Rs*conj(v2R1)
Eqn 1Rs=v1R1*conj(v2Rs)
Eqn SumRsR1=pRsR1+pR1Rs

Eqn sR2=v1Rs*conj(v2R2)
Eqn 2Rs=v1R2*conj(v2Rs)
Eqn SumRsR2=pRsR2+pR2Rs

Eqn 1R2=v1R1*conj(v2R2)
Eqn 2R1=v1R2*conj(v2R1)
Eqn SumR1R2=pR1R2+pR2R1

Eqn Rdelta1=v1Rs*conj(v2Rdelta1)
Eqn elta1Rs=v1Rdelta1*conj(v2Rs)
Eqn mRsRdelta1=pRsRdelta1+pRdelta1Rs

Eqn Rdelta2=v1Rs*conj(v2Rdelta2)
Eqn elta2Rs=v1Rdelta2*conj(v2Rs)
Eqn mRsRdelta2=pRsRdelta2+pRdelta2Rs

pEqn a1Rdelta2=v1Rdelta1*conj(v2Rdelta2)
pEqn a2Rdelta1=v1Rdelta2*conj(v2Rdelta1)
pEqn Rdelta1Rdelta2=pRdelta1Rdelta2+pRdelta2Rdelta1

Eqn elta1R1=v1Rdelta1*conj(v2R1)
Eqn Rdelta1=v1R1*conj(v2Rdelta1)
Eqn mRdelta1R1=pRdelta1R1+pR1Rdelta1   Use this also for Rdelta1+R2, Rdelta2+R1 and Rdelta2+R2 noise

abEqn umRsR1)+abs(pSumRsR2)+abs(pSumR1R2)+abs(pSumRsRdelta1)+abs(pSumRsRdelta2)+abs(pSumRdelta1Rdelta2)+4*abs(pSumRdelta1R1)
Eqn =sqrt(AC.freq)

ADS                                                    ADS
```

Figure 12f. Equations for uncorrelated noise terms. 2-channel complex cross-correlation power computation for each pair-wise noise sources. While each noise source is uncorrelated to another, there are two cross-correlation terms for each pair that are correlated, and so a vector sum could be taken for these two terms as shown above. It did not make a difference to the total power when the sum of magnitudes was taken instead. For the total uncorrelated power, only the noise power magnitudes are added from each pair of noise sources.

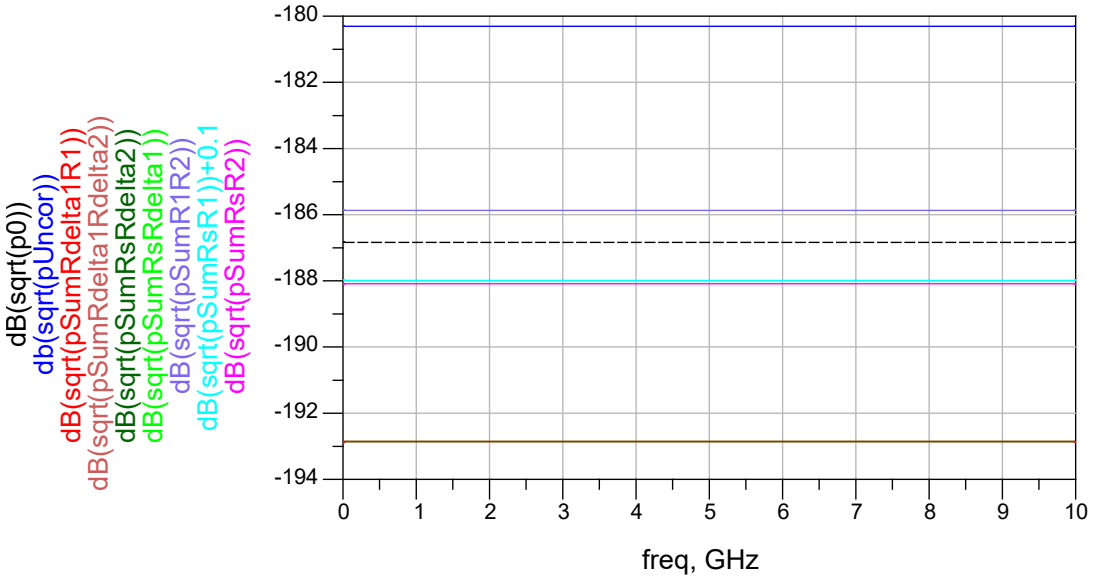


Figure 12g. Uncorrelated noise powers in dBV/Hz. Correlation noise suppression is not included.

Simulation results

The simulation results for the total power, total correlated power, and total uncorrelated power in dBV/Hz are shown below in Figure 13a and 13b. In Figure 13b, the a priori known “constant” 2.071e-19 W correlated power equivalent to thermal -187 dBV/Hz from termination resistors R1 and R2 has been subtracted from the total power to improve the noise floor at low frequencies. This method is suggested to be adopted for low frequencies before the first dip occurs. From the dip onwards, we should not do the subtraction as the noise floor increases by 3 dB. Alternatively, we can store the system noise with 50 50-ohm source during calibration and subtract it from the DUT measurement.

Key observations:

- The phase difference between Rs and Rdiff autocorrelation noise powers is always 180 degrees, independent of the path length mismatch between Channels 1 and 2, so their combined autocorrelation powers are annihilated.
- The autocorrelation power of Rs is real at low frequencies and complex at higher frequencies. However, the magnitude is constant if the path delay mismatch is minor over frequency.
- The phase difference between R1 and R2 autocorrelation noise power is close to 0 degrees at low frequencies. At the first power dip of 8 MHz and at odd harmonic frequencies of the first dip, the phase difference is 180 deg or odd multiples of it, and the autocorrelation noise powers subtract, leading to a dip in the total cross-correlation noise power. The dip frequency is dependent on the cable length and not on their slight mismatch. The longer the cable, the lower the frequency where the dips occur.
- The total cross-correlation noise power is only due to R1 and R2, and of course, any noise that is above the thermal noise from the Source that is being measured. The Source resistance thermal noise from Rs of -187 dBV/Hz is cancelled by the anti-correlated noise from Rdiff.
- Autocorrelation noise powers from Rdelta1 and Rdelta2 are negligible.
- The averaging of the autocorrelation noise power pRs, pRdiff, pRdelta1, pRdelta2, pR1 and pR2 over the number of cross-correlations does not reduce their values as these are either always correlated (always positive sign) or always ant-correlated (always negative sign) terms or may have some fixed phase $\phi(f)$ other than 0 or 180 deg dependent on the skew between the channels.
- Uncorrelated noise terms are produced by cross-correlation of noise voltages between uncorrelated noise sources, and these will reduce by 5*log10(N) dB where N is the number of cross-correlations (and their averaging). Their terms are theoretically computed and plotted in Figure 13a and 3b. A Transient SPICE simulation is required to physically obtain these terms, with cross-correlation postprocessing done in Matlab.
- Uncorrelated noise power terms are produced by cross-correlation of noise voltages of two different resistors or noise sources that are uncorrelated to each other. A two-channel complex cross-correlation power computation is done for each pair-wise noise sources. While each noise source is uncorrelated to another in the pair, there are two cross-correlation terms for each pair that are correlated, and so a vector sum is taken for these two terms as shown in Figure 1f. However, taking the sum of magnitude instead did not make a difference to the total power. For the total uncorrelated power, only the noise power magnitudes are added from each pair of noise sources.

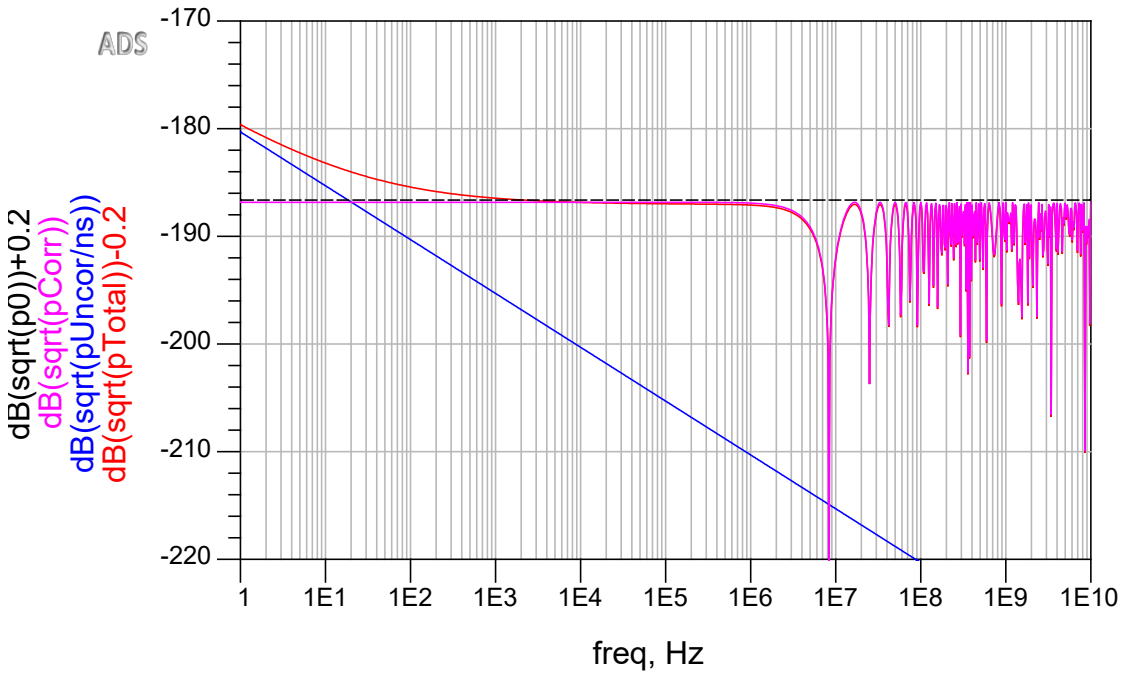


Figure 13a. Correlated (pCorr), Uncorrelated (pUncor), and Total (pTotal) noise power. The noise suppression of uncorrelated power is taken as an ideal case of ns = sqrt(frequency), assuming 1x cross-correlation at 1 Hz and thereafter proportional to the square root of frequency. The 50 ohms thermal floor is p0 at -187 dBV/Hz.

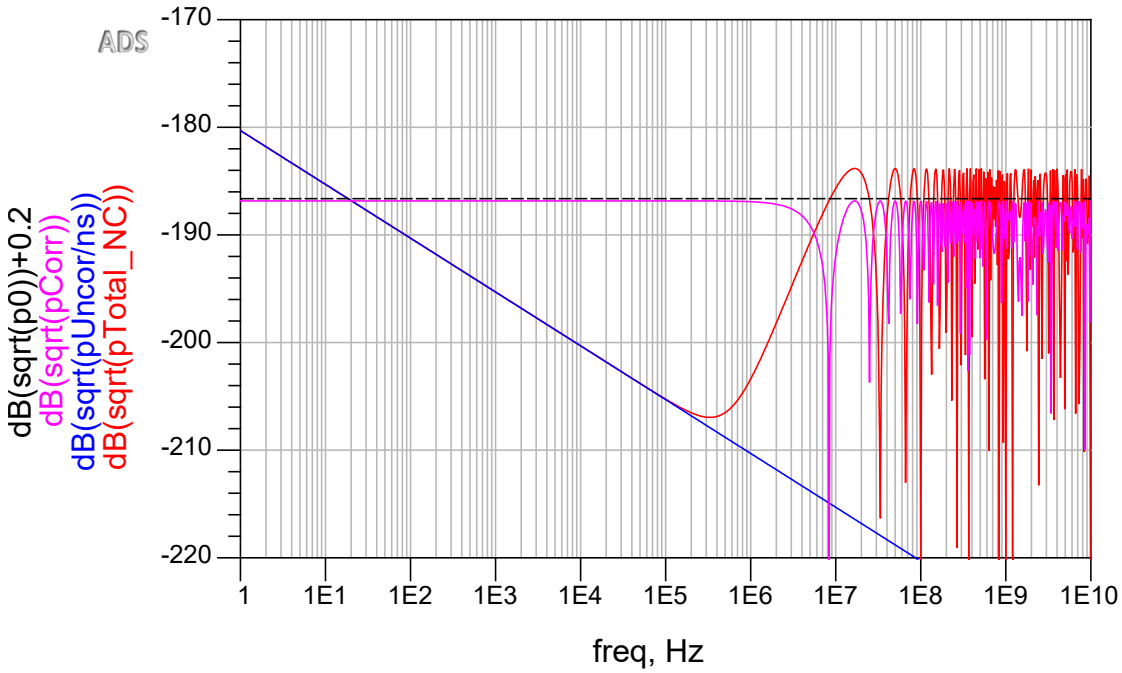


Figure 13b. With noise correction by subtracting R1 and R2 autocorrelated powers that are known a priori (theoretically at low frequencies), the system noise floor (pTotal_NC) can be significantly improved. Beyond the first dip frequency of 8 MHz, the noise floor increases periodically by 3 dB above thermal. The noise floor can be further lowered by subtracting the uncorrelated noise term pUcorr/ns that is also known a priori for a 50-ohm system.

Phase Noise Cross-spectral Collapse

The cross-spectral collapse in phase noise measurements follows the same principles as the baseband noise measurements cross-spectral collapse, but in addition, there is spectral collapse due to the phase noise from the reference oscillators in the two channels. The reference (oscillator) phase noise behaves like the autocorrelation noise from R1 and R2 termination resistors that annihilate each other when their relative phase becomes 180 deg. The cross-spectral collapse in phase noise is investigated using ADS Harmonic Balance (HB) simulation at 5 GHz using a 4-stage broadband Wilkinson Power Divider to split the Source signal for the Channel 1 and Channel 2 paths of the cross-correlation setup. The time-domain cross-correlation is done in the frequency domain by taking the products of the tone-based voltage terms between the two channels, with channel 2 terms being conjugated.

Simulation setup using harmonic balance

Phase noise spectrum is typically the upper or lower sideband frequency domain spectrum which is normalized to dBc/Hz units relative to the carrier power. During the spectrum measurement, the AM noise is removed by normalizing the signal to the envelope. In the N5511A, the computed Phase noise spectrum is the Power Spectral Density (PSD) measurement of the actual demodulated phase of the carrier using a 2-channel cross-correlation technique. In the ADS HB simulation shown in Figure 14, the phase noise is generated at each noise source using upper and lower sideband 1-tone signals with opposite polarities. For a 50 ohms resistor, the magnitude of each sideband tone is -190 dBV rms (-177 dBm/Hz integrated over 1 Hz bandwidth). At each of the Channel 1 and 2 output ports, the downconverted voltages of these two sidebands are summed to produce v1(f) and v2(f), respectively, where f is the offset frequency of the sidebands from the carrier. Then their cross-correlation power is computed by taking the product v1(f)*conj[v2(f)]. When v1(f) and v2(f) are from the same noise source, the product is the autocorrelation power. If they are from different noise sources, then the product is a cross-correlation noise power. The gain in the system is calibrated so that the Channels 1 and 2 cross-correlation phase noise from the 50 ohms Source resistor Rs produces -190 dBV rms autocorrelation voltage (for 50 ohms load) with the upper and lower sideband tones of Rs also set to -190 dBV rms.

The thermal noise contribution from various sources is measured in dBV/Hz in the Y-axis in the Harmonic Balance simulation results below. The 50 ohms thermal phase noise and AM noise components are -190 dBV/Hz each. After subtracting the dBV (rms) Carrier voltage, the corresponding phase noise in dBc/Hz is obtained if needed. The Carrier is not included in the simulations. Only the 50-ohm source resistance noise is included. A 4-stage Wilkinson Power Divider splits the Source noise into two channels that include equal-length transmission lines, noisy 50-ohm terminating resistors, followed by quadrature down conversion to 0Hz baseband. The two baseband signals, one from each channel, are cross-correlated using user-defined cross-spectral computation equations in the HB simulation. AM and PM thermal noise can be enabled in the source Rs, and separately also for all other resistors. The AM noise level can be independently set relative to the phase noise in the source Rs.

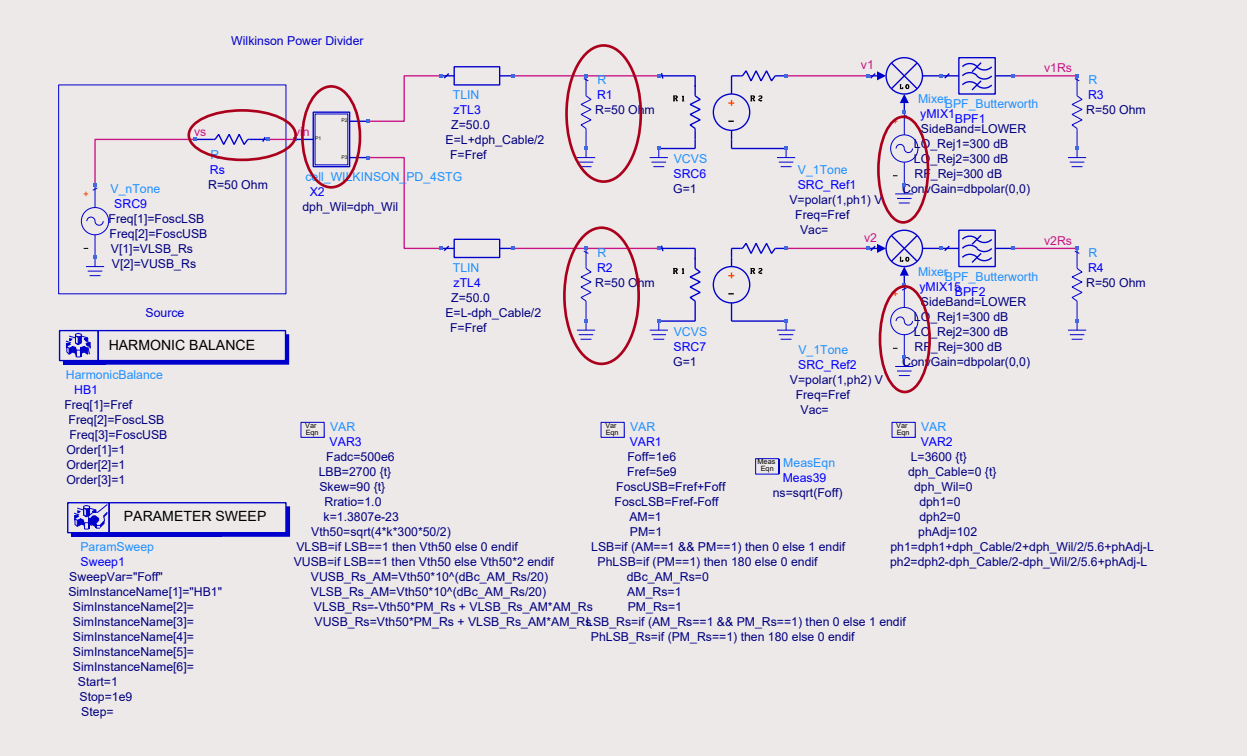


Figure 14. ADS HB simulation setup. Circled components contribute noise in the HB simulation. Noise at an offset from the carrier is modeled as 4 tones, 2 for AM and 2 for PM.

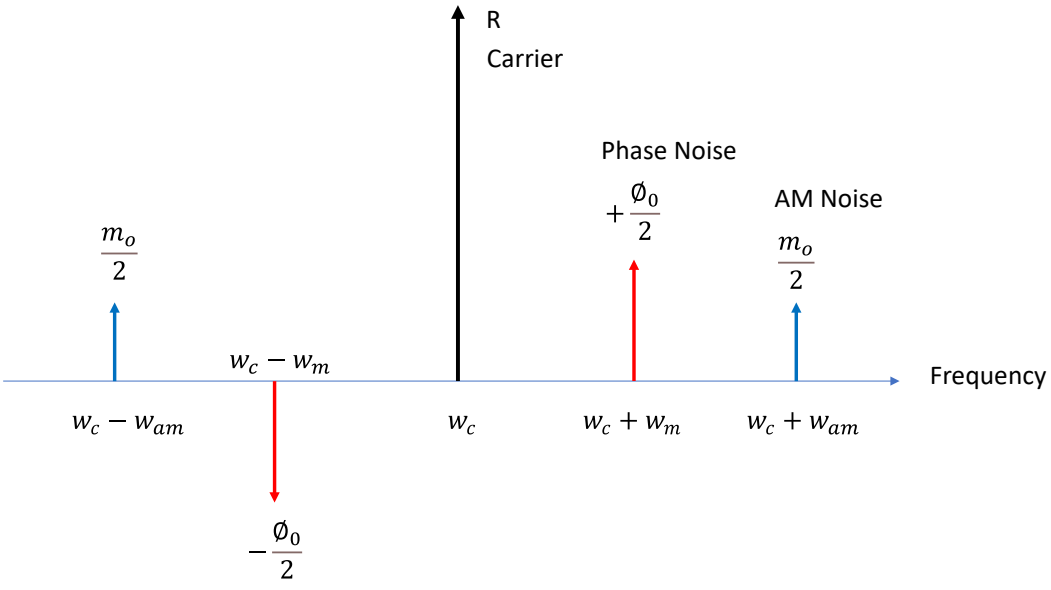


Figure 15. AM and PM spurs with modulation frequencies w_{am} and w_m respectively

The rms voltage of each of the four tones in the above figure, when considered for thermal noise at 300K, is -190 dBV/Hz when their individual powers are considered in a 1Hz bandwidth each. The total rms voltage in a 1Hz bandwidth is -187 dBV/Hz and is the rms value of the combined AM and PM noise voltages. If the carrier voltage is 0 dBV, then the single-sided spectrum-based phase noise $L_v(\mathrm{f}) = \frac{\emptyset_0^2}{4}$ is -190 dBc/Hz from the thermal noise based on the ratio of the sideband power to the carrier power. The PSD of phase noise $S_\emptyset(f_m) = \frac{\emptyset_0^2}{2}$ is due to the contribution of both the upper and lower sidebands and the rms value of their combined voltage is -187 dBV/Hz.

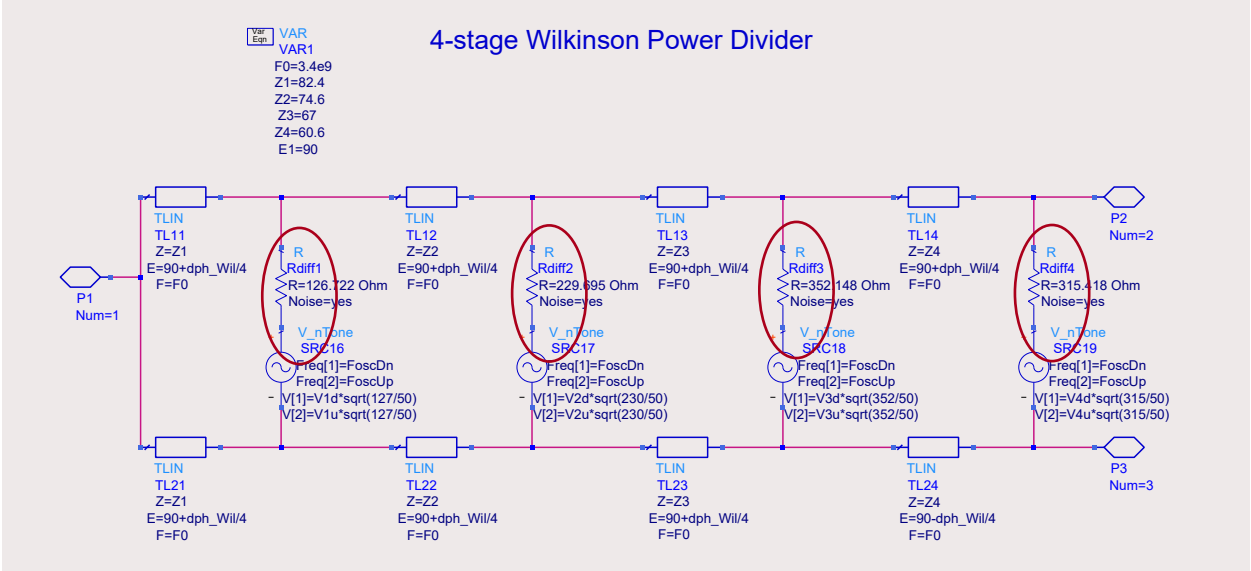


Figure 16. Measuring the noise of each resistor in the 4-stage Wilkinson Power Divider

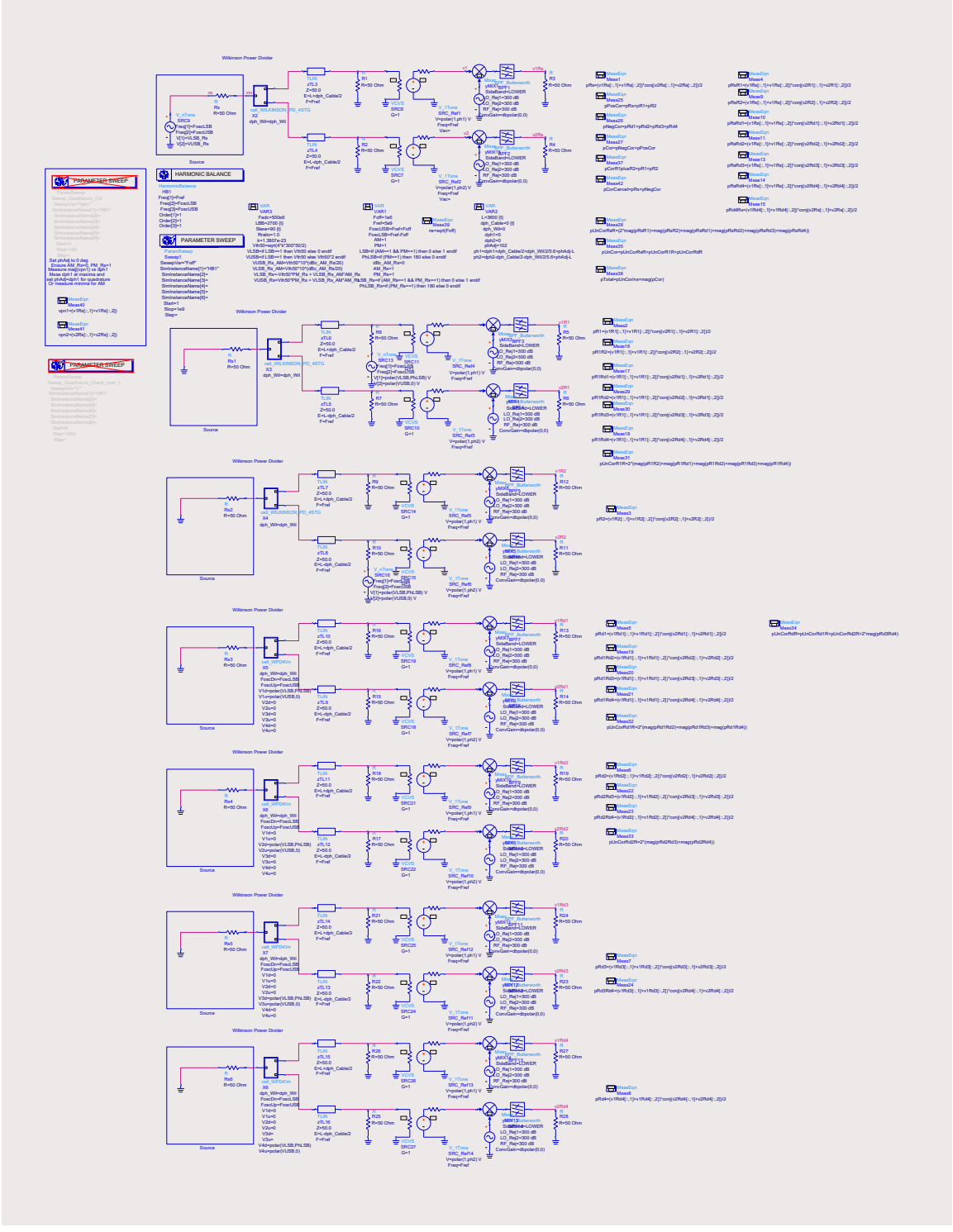


Figure 17. There are totally 7 resistors contributing noise and therefore the circuit is repeated 7 times with just one voltage source enabled in each for the resistor under consideration. Phase noise of the references Ref1 and Ref2 are simulated separately (not shown).

The Ref1 and Ref2 phases are initially calibrated to get phase quadrature that is indicated by the AM conversion gain of the phase detectors having a sharp dip as shown in Figure 18 below. The AM conversion suppression is observed to be 21.2 dB for +-5 deg quadrature phase error. In later simulations, the Source AM noise will be set to 21.2 dB above PM noise and the total noise results will be checked for quadrature errors of [0,0], [-5,5], [5,-5], [-5,-5] and [5,5] deg for the two channels. When the quadrature errors are of opposite polarities of [-5,5] and [5,-5], the AM and PM noise should cancel for the case of AM noise being 21.2 dB larger than PM noise.

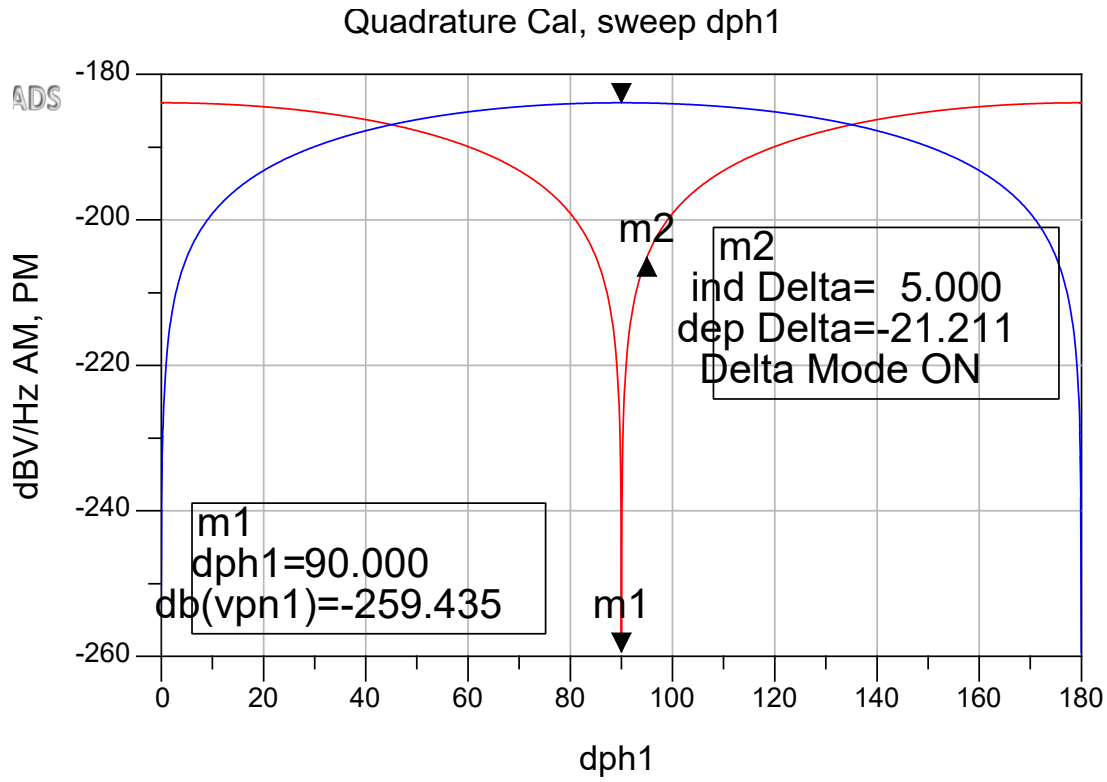
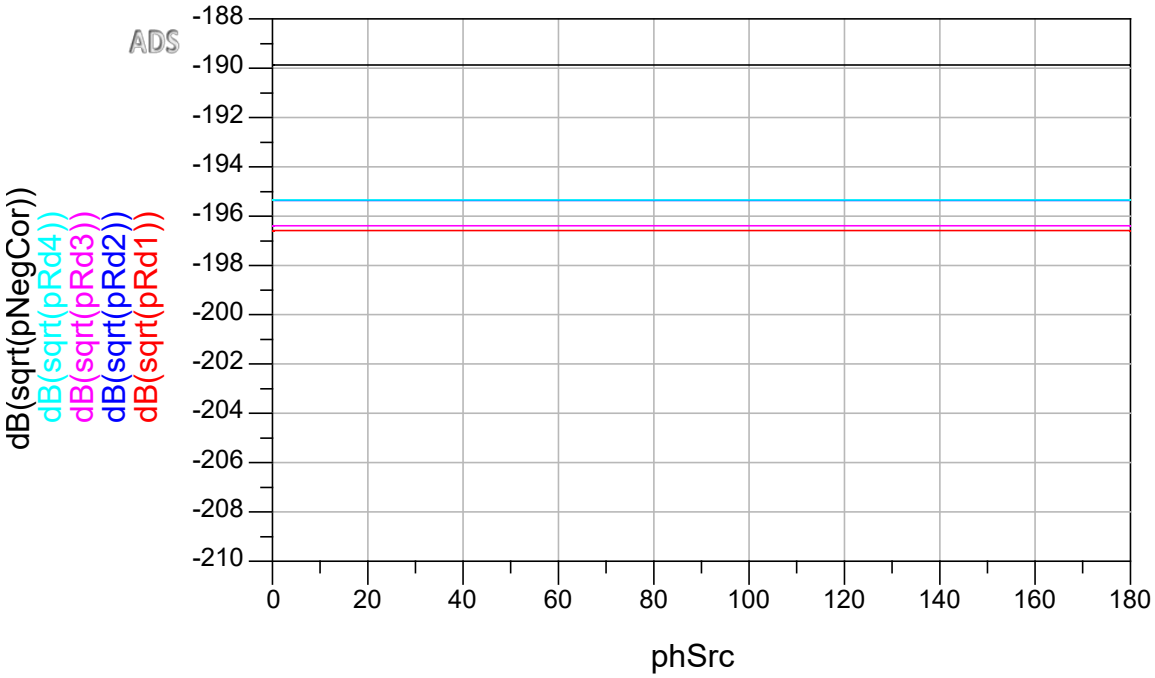


Figure 18. AM and PM Gain vs RF-LO relative phase. The Ref1 and Ref2 phase are initially calibrated for phase quadrature.

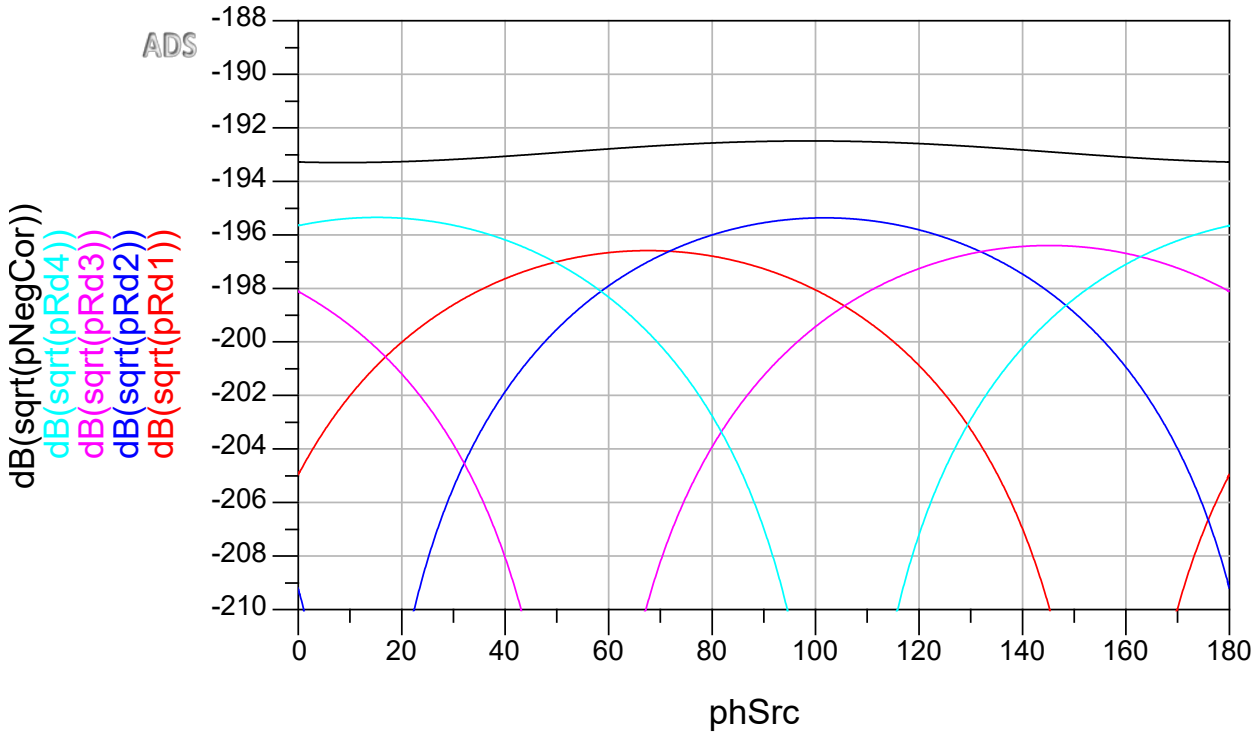
Simulation results

Anti-correlated noise of Wilkinson Power Divider

The anti-correlated noise power of the four resistors in the WPD and the total anti-correlated noise power are shown below when both AM and PM noise are enabled in each resistor. The power in dBV/Hz is plotted over the phase of Ref1 and Ref2 LO signal of the Phase Detector. Because both AM and PM noise are enabled, the anti-correlated powers are constant over the LO phase.



When only AM or only PM noise is enabled in each resistor, the anti-correlated powers vary over the LO phase. The case for only PM noise is shown below. As the phase of the LO is swept relative to that of noise in each resistor is swept, the down converter conversion gain for AM and PM detection varier with the peak of one falling on the null of the other. The peaks and nulls of each resistor occur at different LO phases due to delays between them from the connecting transmission in-between them.



In the remaining sections of this paper all the simulation results include equal AM and PM noise in the WPD resistors and the R1 and R2 terminating resistors. The AM noise level in the Source resistor Rs can be set to a dB value relative to the PM noise.

Correlated noise cancellation of termination resistors

Equal AM and PM noise from all resistors, perfect phase quadrature.

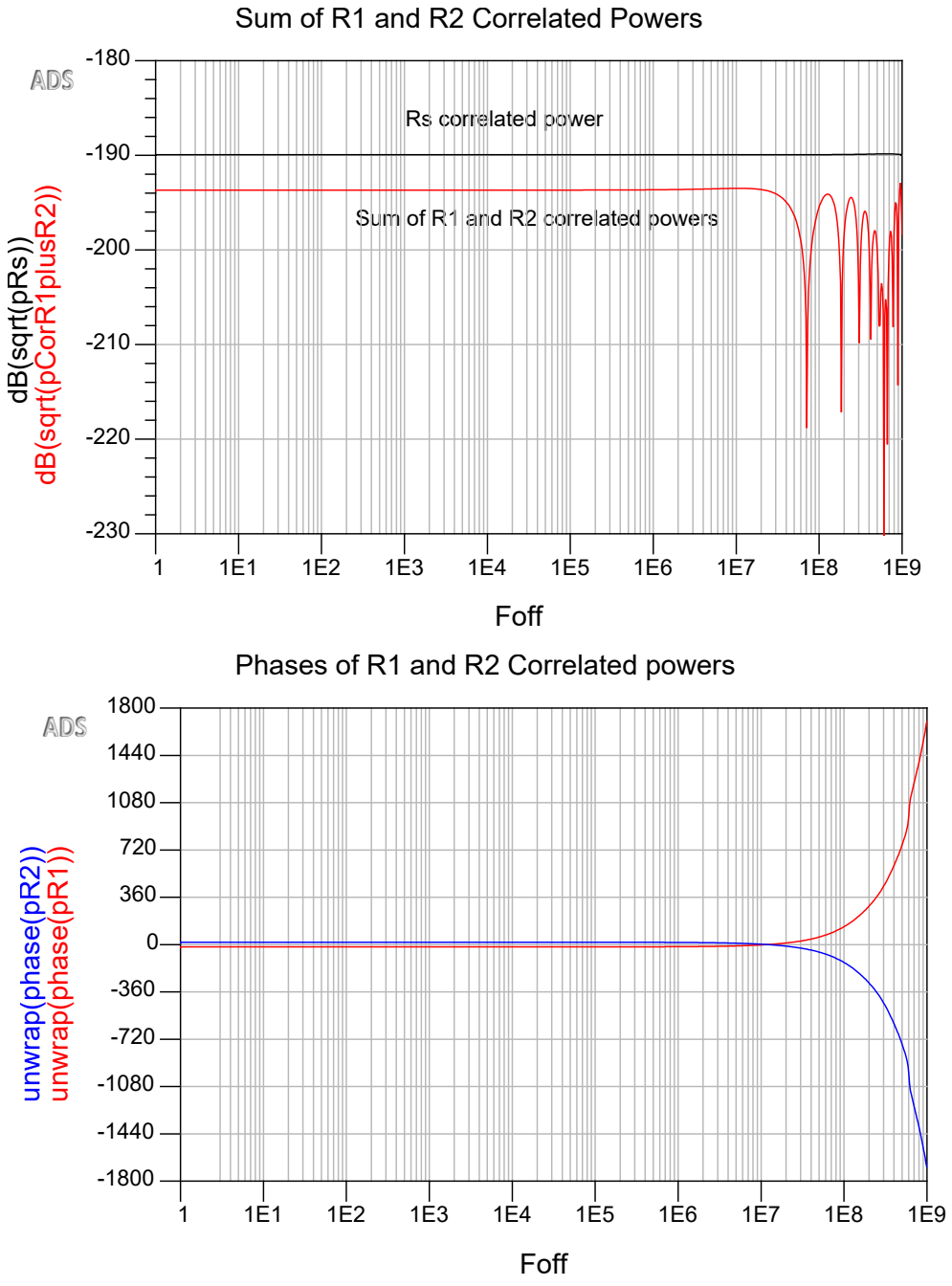


Figure 19. Total Correlated noise power from R1 and R2, and Phase of R1 and R2 correlated noise powers

Source resistor noise cancellation by WPD anti-correlated noise

Equal AM and PM noise from all resistors, perfect phase quadrature.

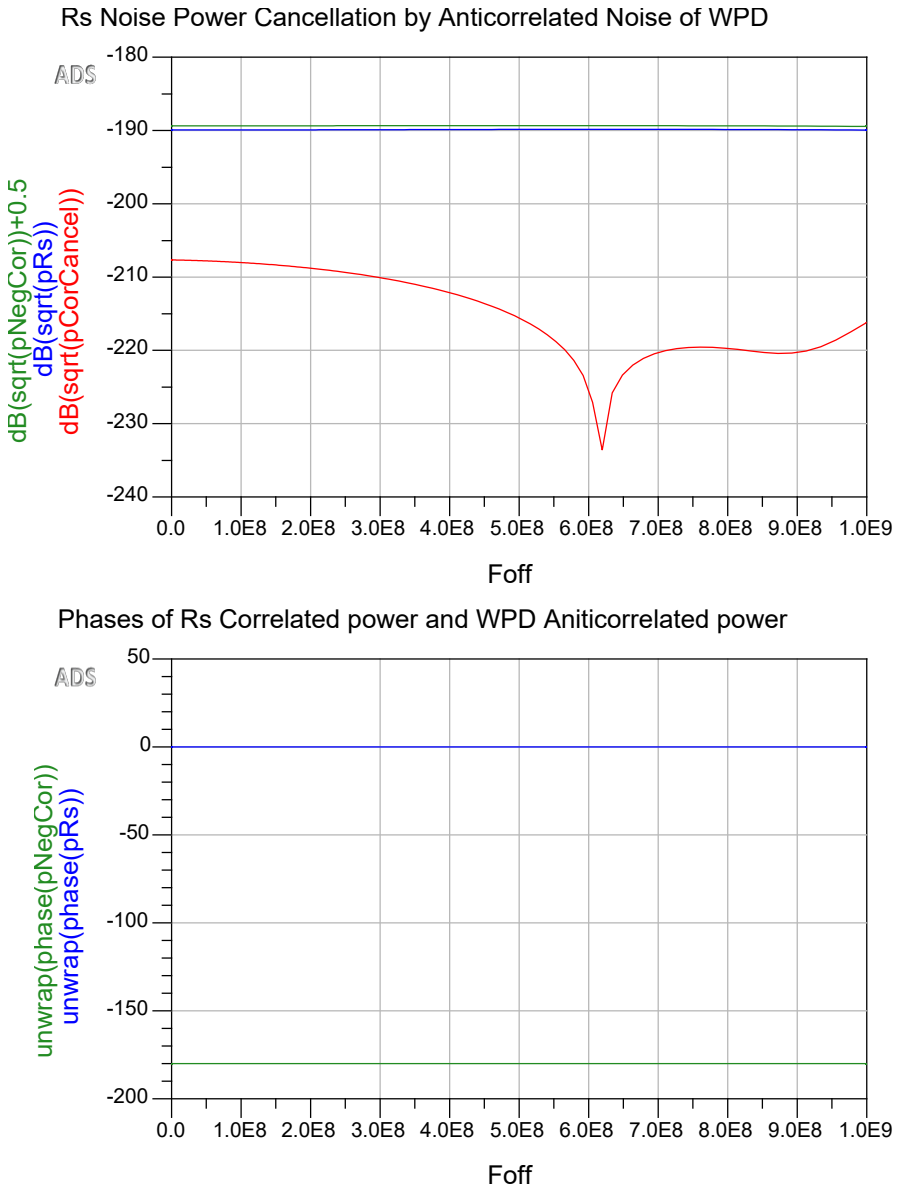


Figure 20. Top: Cancellation of Rs correlated noise power by Anti-correlated noise power of WPD. Red trace is the sum which is reduced by 10 to 20 dB compared to the original power level of Rs (blue trace).

Bottom: 180 deg phase difference between Rs correlated power and WPD anti-correlated power

Total correlated and uncorrelated noise

In the following simulation, the powers are defined as follows:

- pRs = correlated power of Source Rs with equal AM and PM noise.
- pNegCorr = anti-correlated power of WPD
- pCorR1plusR2 = combined correlated power of R1 and R2. Their imaginary parts cancel each other.
- pCorr = sum of all correlated and anti-correlated power
- pUncor/ns = total uncorrelated power divided by correlation gain of ns
- pTotal = sum of all powers

Other settings:

- Equal AM and PM noise from all resistors, at -190 dBV/Hz each
- Perfect quadrature phase.

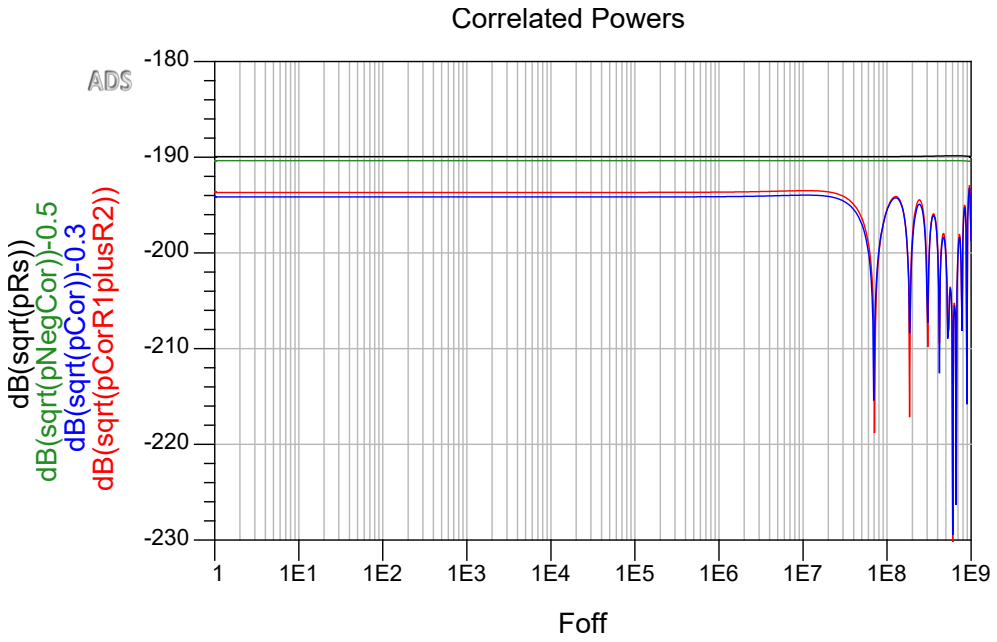


Figure 21. Green and blue traces cancel, so total correlated power (blue trace) is equal to the red trace (sum of R1 and R2 correlated powers)

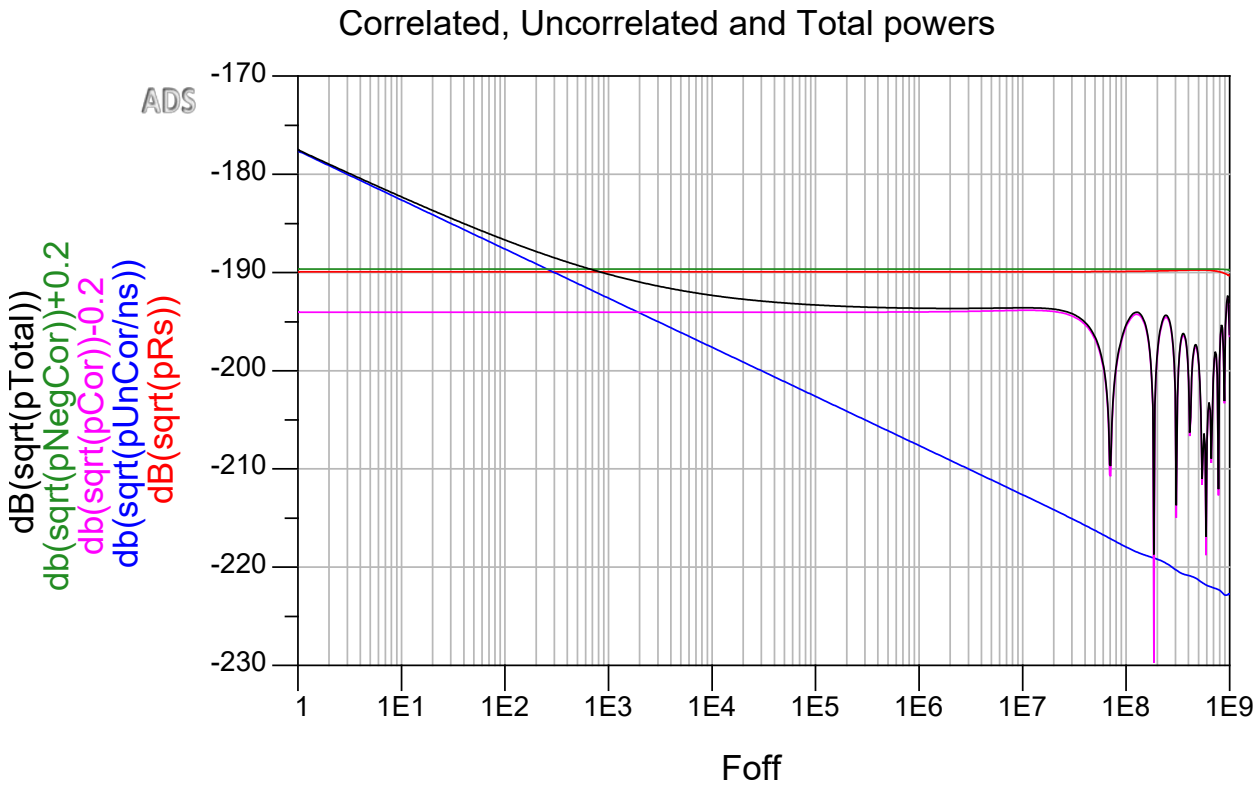


Figure 22. AM is fully suppressed and does not add to the total noise. Total noise of -193 dBV/Hz is due to R1 and R2 at large offsets. Rs power at -190 dBV/Hz due to phase noise part only as AM part is fully suppressed. Rs noise is cancelled by Rdif anticorrelated noise.

Simulation with quadrature phase errors

AM noise is set 21.2 dB above PM noise in Rs. Equal AM and PM noise enabled in other resistors. When quadrature phase error is set to 5 deg, the Phase detector's AM suppression degrades to 21.2 dB, bringing the Source resistor AM noise to same level of the PM noise after down conversion to baseband.

Phase error +5, +5 deg

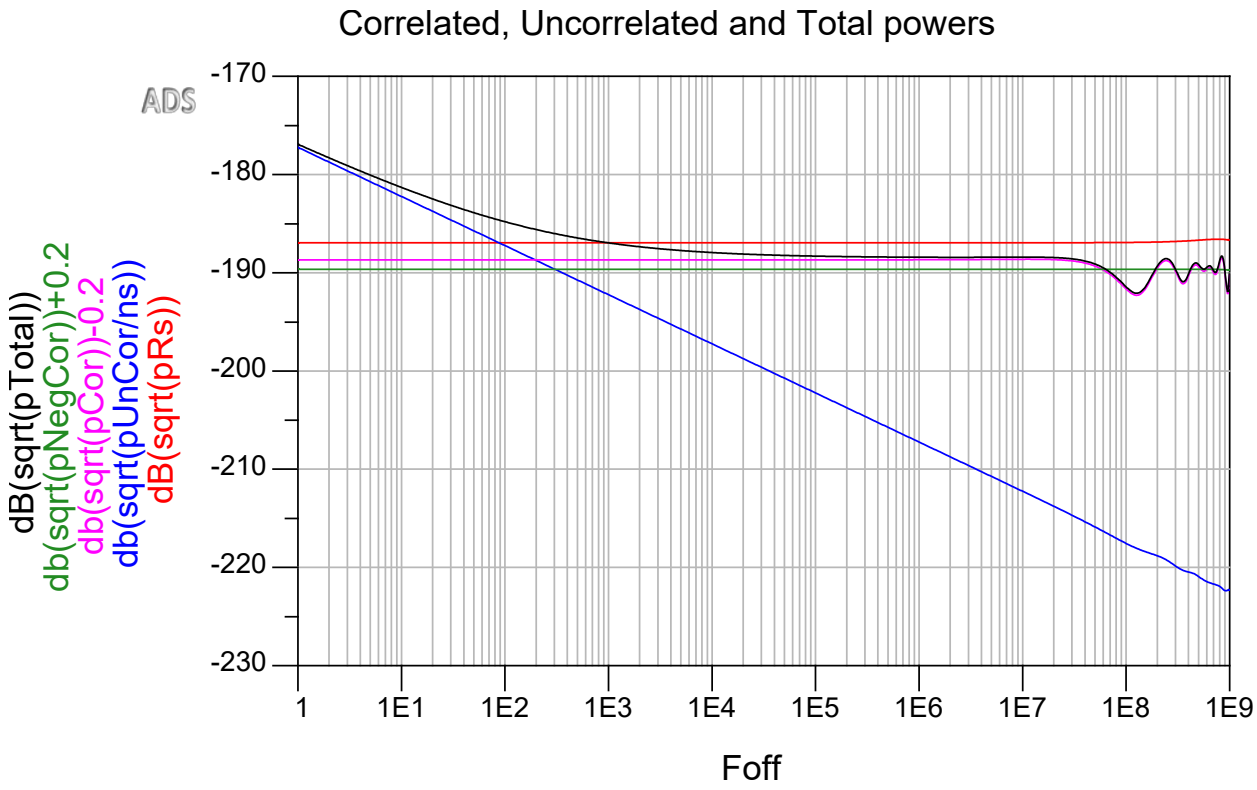
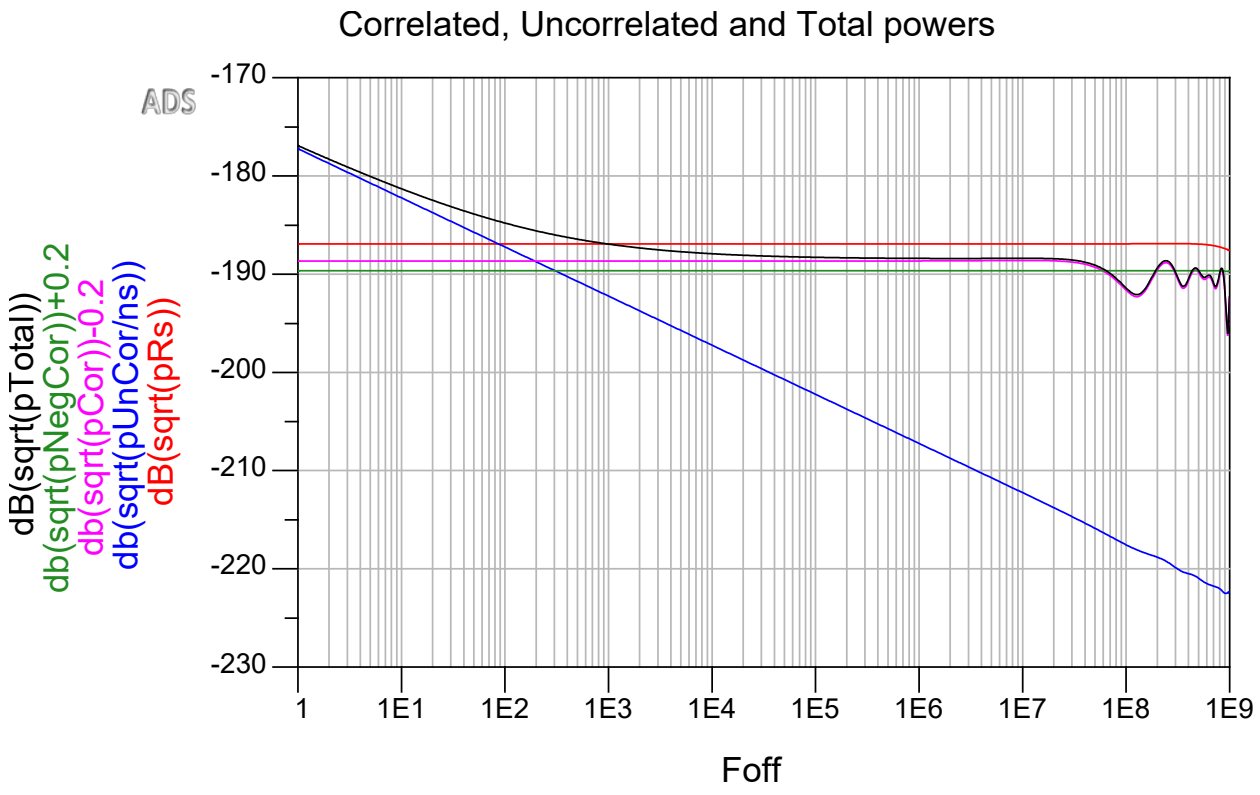
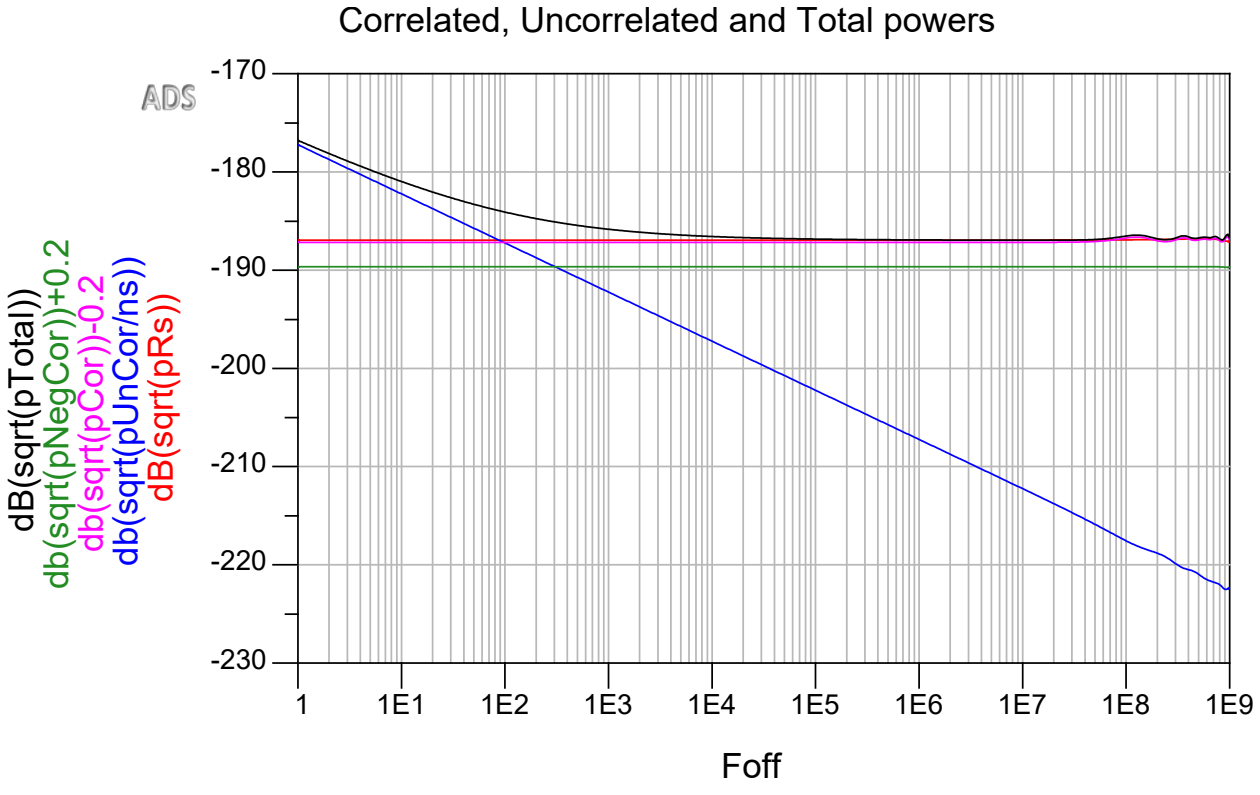


Figure 23. Suppressed AM is equal to PM noise in Rs and add to the total noise. Rs noise increases to -187 dBV/Hz. There is no AM-PM cancellation. Rs suppressed AM power is -190 dBV/Hz and it increases total noise to -188 dBV/Hz.

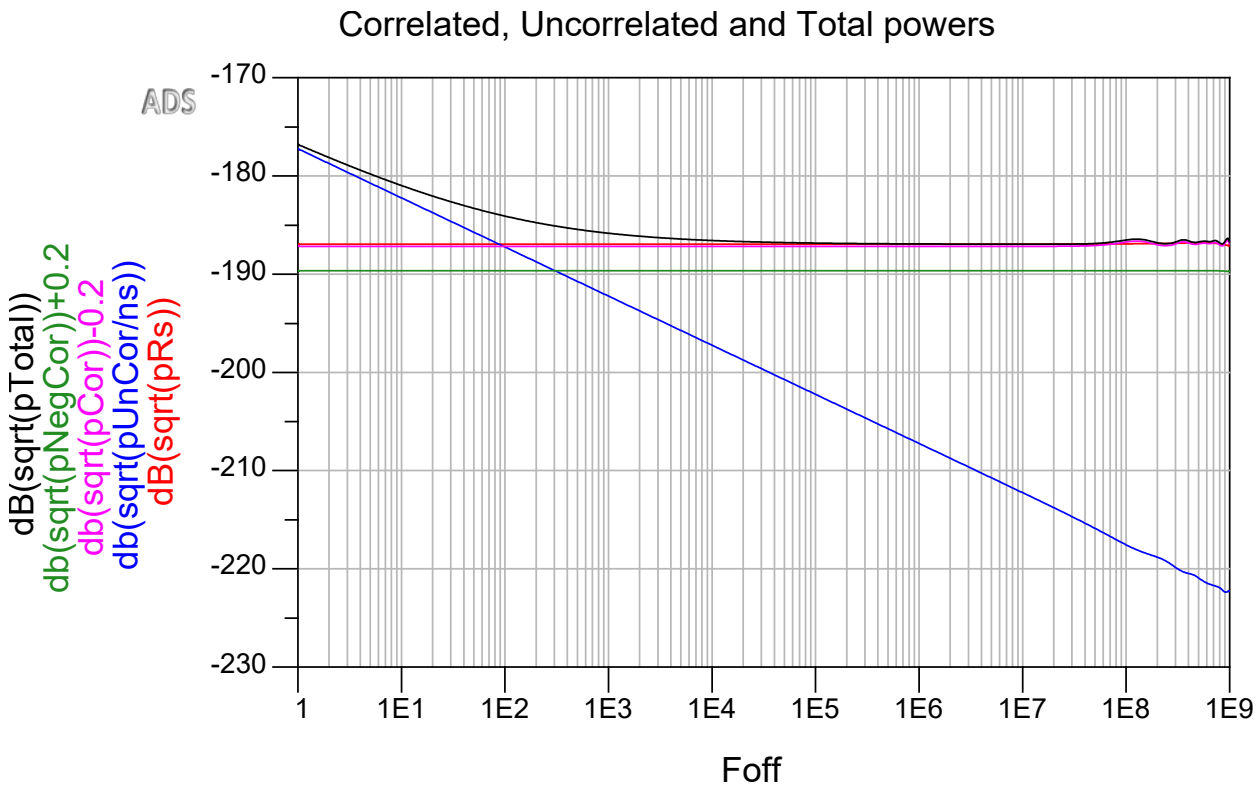
Phase error -5, -5 deg



Phase error -5, +5 deg

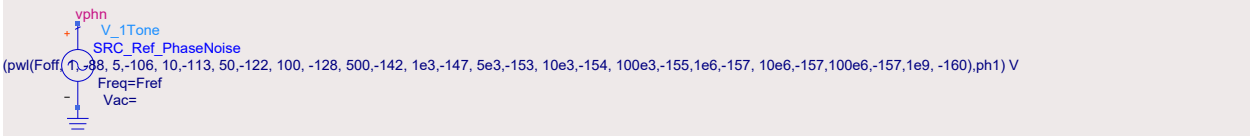


Phase error +5, -5 deg



Reference phase noise cross-spectral collapse

The cross-spectral collapse of the Reference phase noise due to leakage from one channel to the other has been simulated with the results shown below. In this simulation the channel-channel isolation was set to 35 dB, and the E8257D PSG phase noise at 5 GHz was used for the reference. The Reference phase noise model in ADS is set up as below and shown as the red trace in the simulation results below.



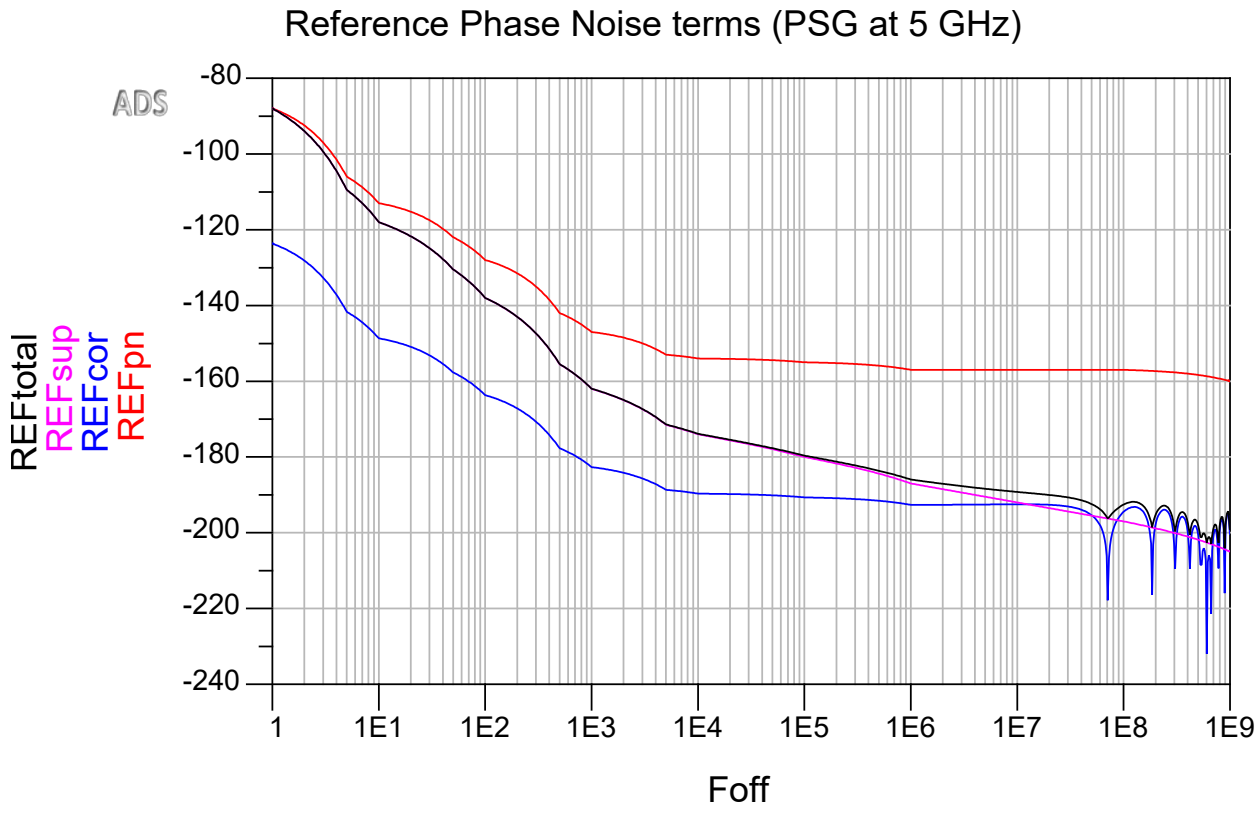


Figure 24. Cross-spectral collapse of noise floor from reference phase noise

Red trace is the Reference phase noise model for Ref1 and Ref2.

Pink trace is the same phase noise with suppression due to cross correlation that increases with offset frequency.

Blue trace corresponds to the $\boldsymbol{kN1N1^{*}e^{-j\Delta\theta n} + kN2N2^{*}e^{+j\Delta\theta n}}$ term of Eq 16 i.e., cross correlation of the Ref1 and its leakage to channel 2, summed with that due to Ref2 cross correlated to its leakage to channel 1. Whenever the phase shift of the leakage path is an odd multiple of 90 deg, there is a spectral dip due to collapse.

Black trace is the total cross spectrum i.e., the sum of the Pink and the Blue traces, and therefore has weaker spectral collapse.

Calculation of the spectral terms in ADS:

```
EqnCorR1R2=-193
Eqnol_WPD_Mixer=35
EqnFpn=dB(vphn[::,5])
Eqnr=REFpn+dB(sqrt(pCorR1plusR2))-pCorR1R2-Isol_WPD_Mixer
Eqnfsup=REFpn-5*log(Foff)
Eqnal=10*log10(pow(10,REFcor/10)+pow(10,REFsup/10))
```

Measured cross-spectral collapse

To see cross-spectral collapse in the phase noise measurement the easiest approach is to degrade the isolation between the two channels by substituting a RPD in place of a WPD and use Ref1 and Ref2 with sufficiently high phase noise compared to that of the DUT. The reference noise leaking through the RF port of the Phase Detectors to the other channel via the RPD will produce the cross-spectral collapse as explained in the previous sections and shown in the simulations. The DUT noise (AM and PM) must be low compared to the reference leakage noise terms across the channels so that the measured noise is dominated by those from the references. Dips in the measured phase noise spectrum indicate cross spectral collapse. As explained in previous sections, it occurs even though the Ref1 and Ref2 sources are uncorrelated. The offset frequency of the spectral dips will change with the delay in the channel i.e., the lengths of the RF cable used after the RPD.

For the N5511A 2-channel phase noise measurements shown below at 4.8 GHz in Figure 25, the DUT was a H4 signal source from Keysight with extremely low phase noise. Very long (~1m) RF cables were used after the RPD. Three spectral dips are clearly seen.

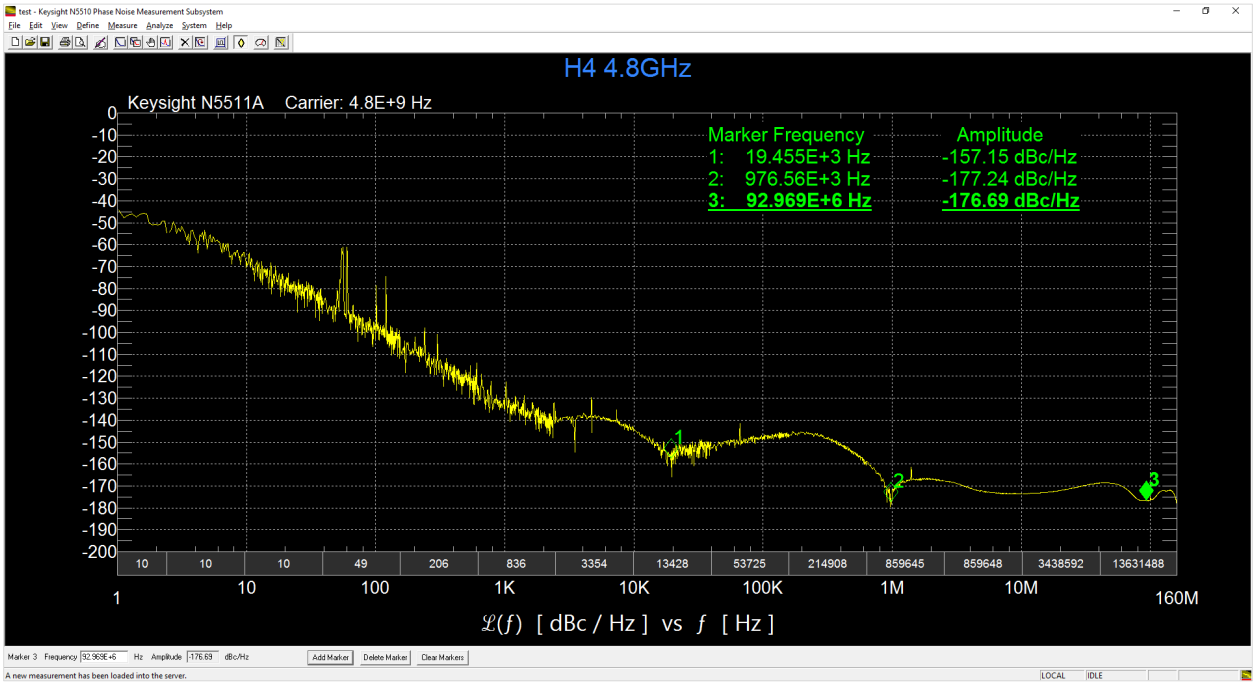


Figure 25. Cross-spectral collapse with long RF cables

In the phase noise measurement results of Figure 26, very short (6 cm) RF cables were used after the RPD. Only one spectral dip is seen around 150 MHz offset.

Measurement using a WPD with long RF cables is shown in Figure 28. Cross-spectral collapse is not visible with long RF cables due to improved isolation when WPD is used.

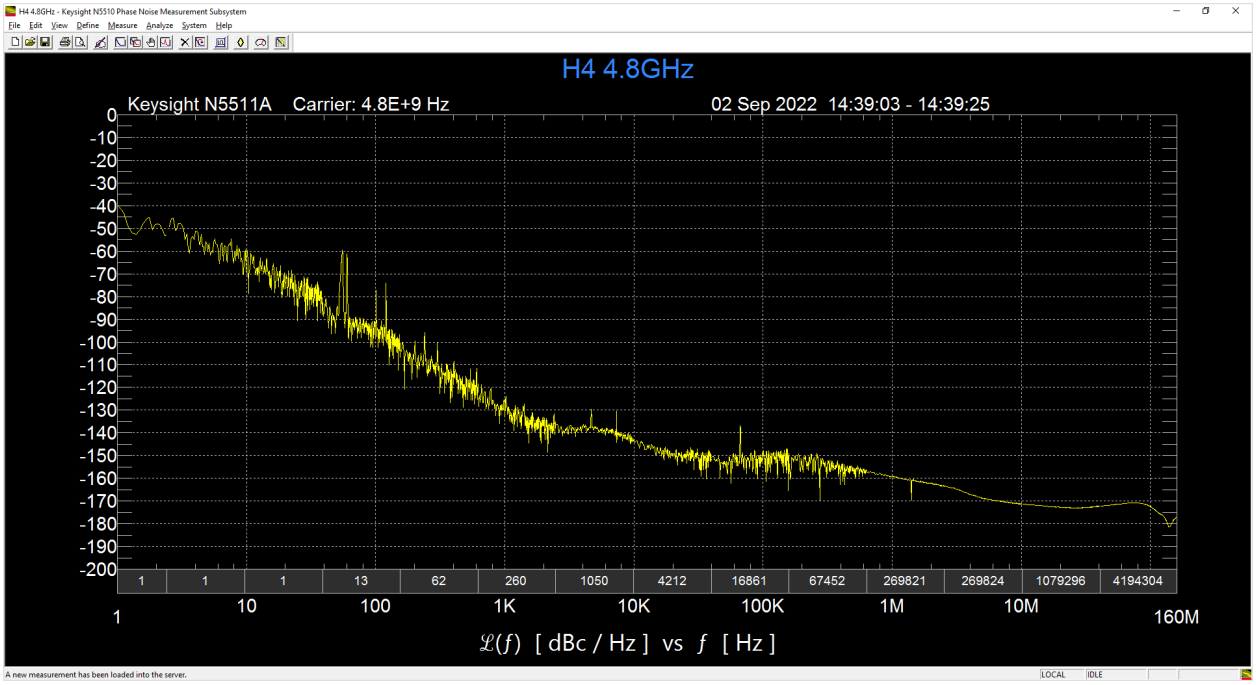


Figure 26. Cross-spectral collapse with short RF cables

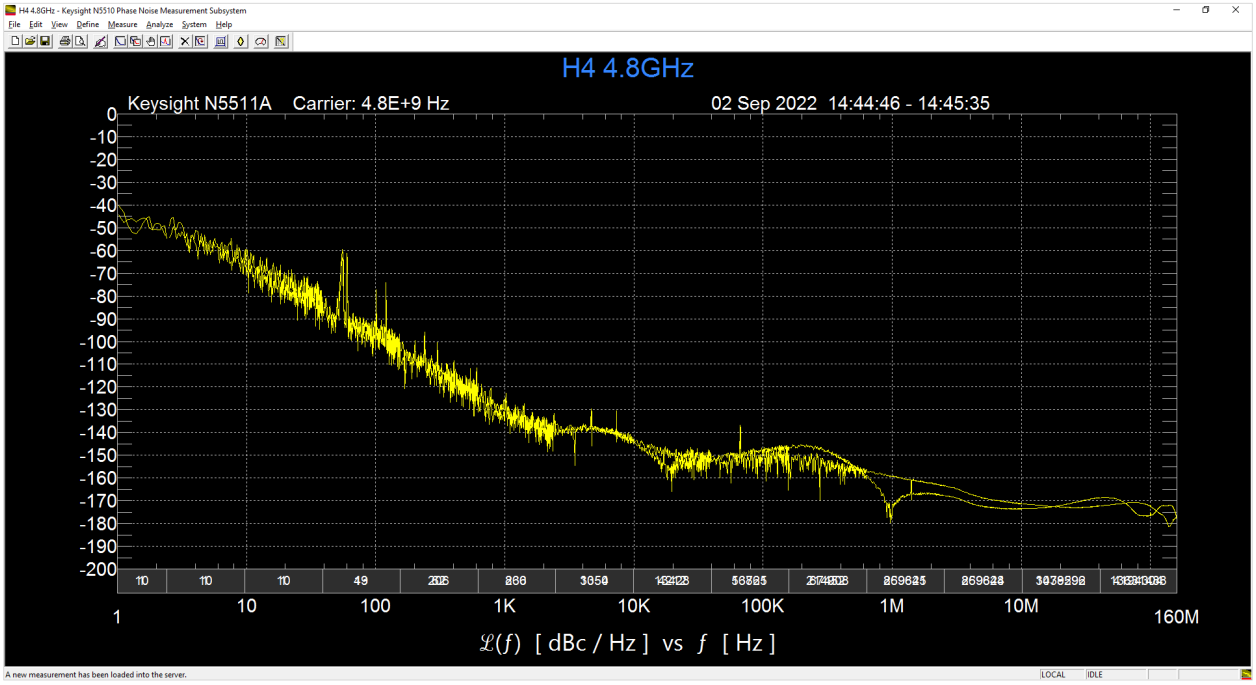


Figure 27. Overlaid plots of RPD with long and short RF cables

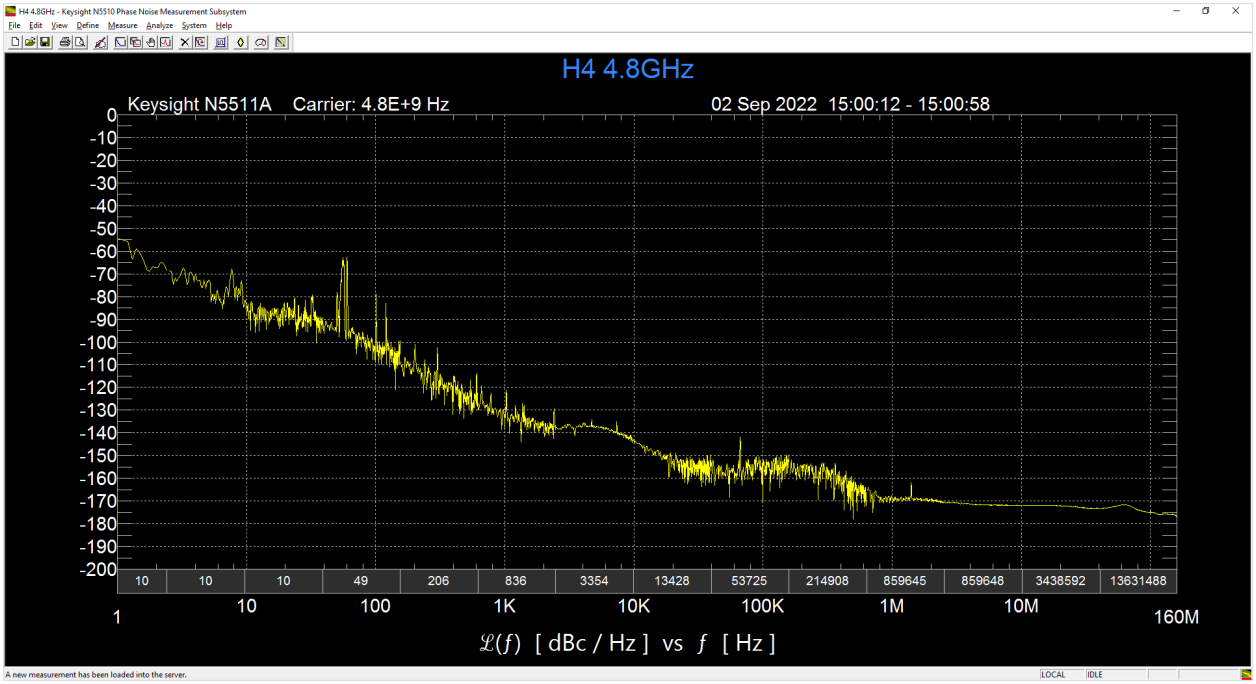


Figure 28. Absence of cross-spectral collapse with long RF cables due to improved isolation when WPD is used

AM PM Cancellation of Carrier Noise

In the presence of quadrature phase error in the phase detector, the AM in the Source is not suppressed sufficiently and produces a bias or error in the cross-correlation based phase noise power measurement. Depending on the signs of the quadrature phase detector error in the two channels, the AM of the source can produce correlated or anti-correlated power that add to or subtracts from the phase noise power. Under some conditions the AM and PM noise can cancel completely. It is generally assumed that the AM and PM modulation noise waveforms in the Source are uncorrelated, and the analysis below considers this aspect. However, we also consider later the case when the two modulating waveforms are correlated i.e., originate from a common noise signal.

An RF carrier (source) with both AM and PM noise is given by the sum of three terms:

$$s(t) = \cos(w_c t) + n_{am}(t)\cos(w_c t) - n_\theta(t)\sin(w_c t)$$

where the first term $\cos(w_c t)$ is the carrier signal, and the other two terms are the AM and PM noise. $n_{am}(t)$ and $n_\theta(t)$ are the baseband representation of the AM and PM noise around the carrier frequency w_c. The source signal $s(t)$ which is dominated by the carrier signal $\cos(w_c t)$, is phase locked to reference signals $-\sin(w_c t + \Delta\theta 1)$ and $-\sin(w_c t + \Delta\theta 2)$ in each channel. The reference signals are in phase quadrature with the source $s(t)$ but have quadrature phase errors $\Delta\theta 1$ and $\Delta\theta 1$ in channels 1 and 2 respectively. The quadrature errors occur due to RF and LO self-detection in the down converter Phase detector. The signals at the Phase detector outputs of the two channels are

$$v1(t) = -s(t) \cdot \sin(w_c t + \Delta\theta 1)$$

$$v2(t) = -s(t) \cdot \sin(w_c t + \Delta\theta 2)$$

Considering only the baseband terms of the down conversion mixing operation without DC term, and for small quadrature phase errors, the above equations simplify to

$$v1(t) = n_\theta(t) + \Delta\theta 1 \cdot n_{am}(t)$$

$$v2(t) = n_\theta(t) + \Delta\theta 2 \cdot n_{am}(t)$$

The above equations show the contamination of the phase noise $n_\theta(t)$ by the AM noise $n_{am}(t)$ in the signals $v1(t)$ and $v2(t)$. The PSD $P(f)$ of the phase noise is mathematically the Fourier transform of the cross-correlation of $v1(t)$ and $v1(t)$. It is directly computed in frequency domain using the cross spectrum

$$P(f) = V1(f) \cdot V2(f)^*$$

where the capital letters denote the Fourier transform of the corresponding lower-case time signals. Dropping the frequency f for simplicity and expanding the above equation,

$$P = [N_\theta + \Delta\theta 1 \cdot N_{am}] \cdot [N_\theta + \Delta\theta 2 \cdot N_{am}]^*$$

$$\boldsymbol{P = N_\theta N_\theta^* + \Delta\theta 1 \Delta\theta 2 N_{am} N_{am}^* + \Delta\theta 2 N_\theta N_{am}^* + \Delta\theta 1 N_\theta^* N_{am}} \quad \textbf{[1]}$$

where N_θ is the FT of $n_\theta(t)$, and N_{am} is the FT of $n_{am}(t)$.

N_θ and N_{am} are the baseband complex representation of the frequency-domain PM and AM noise respectively.

Case 1: $\boldsymbol{n_{am}(t)}$ and $\boldsymbol{n_{\theta}(t)}$ are uncorrelated

As $\boldsymbol{n_{am}(t)}$ and $\boldsymbol{n_{\theta}(t)}$ are uncorrelated, expected values of $N_{\theta}N_{am}{}^{*}$ and $N_{\theta}{}^{*}N_{am}$ in Eq 1 are 0 while they are complex too. The other two terms $N_{\theta}N_{\theta}{}^{*}$ and $N_{am}N_{am}{}^{*}$ are always positive real and they either add or subtract based on the polarities of $\Delta\theta 1$ and $\Delta\theta 1$ as below.

All the four combinations of the phase error polarities are considered:

a) sign[$\Delta\theta 1, \Delta\theta 2$] = [+,+] or [-,-]

$$P = N_{\theta}N_{\theta}{}^{*} + |\Delta\theta 1\Delta\theta 2|N_{am}N_{am}{}^{*}$$

b) sign[$\Delta\theta 1, \Delta\theta 2$] =[+,-] or [-,+]

$$P = N_{\theta}N_{\theta}{}^{*} - |\Delta\theta 1\Delta\theta 2|N_{am}N_{am}{}^{*}$$

Case 2: $\boldsymbol{n_{am}(t)}$ and $\boldsymbol{n_{\theta}(t)}$ are 100% correlated or anti-correlated

As $\boldsymbol{n_{am}(t)}$ and $\boldsymbol{n_{\theta}(t)}$ are 100% correlated, expected values of $N_{\theta}N_{am}{}^{*}$ and $N_{\theta}{}^{*}N_{am}$ in Eq 1 are real and equal. If anti-correlated, they are negative. The other two terms $N_{\theta}N_{\theta}{}^{*}$ and $N_{am}N_{am}{}^{*}$ are always positive real. For the 100% correlated case,

a) sign[$\Delta\theta 1, \Delta\theta 2$] = [+,+]

$$P = N_{\theta}N_{\theta}{}^{*} + |\Delta\theta 1\Delta\theta 2|N_{am}N_{am}{}^{*} + (\theta 1 + \Delta\theta 2)N_{\theta}N_{am}{}^{*}$$

b) sign[$\Delta\theta 1, \Delta\theta 2$] = [-,-]

$$P = N_{\theta}N_{\theta}{}^{*} + |\Delta\theta 1\Delta\theta 2|N_{am}N_{am}{}^{*} - |\theta 1 + \Delta\theta 2|N_{\theta}N_{am}{}^{*}$$

c) sign[$\Delta\theta 1, \Delta\theta 2$] =[+,-] or [-,+]

$$P = N_{\theta}N_{\theta}{}^{*} - |\Delta\theta 1\Delta\theta 2|N_{am}N_{am}{}^{*} \pm |\theta 1 - \Delta\theta 2|N_{\theta}N_{am}{}^{*}$$

For the 100% anti-correlated case,

a) sign[$\Delta\theta 1, \Delta\theta 2$] = [+,+]

$$P = N_{\theta}N_{\theta}{}^{*} + |\Delta\theta 1\Delta\theta 2|N_{am}N_{am}{}^{*} - (\theta 1 + \Delta\theta 2)N_{\theta}N_{am}{}^{*}$$

b) sign[$\Delta\theta 1, \Delta\theta 2$] = [-,-]

$$P = N_{\theta}N_{\theta}{}^{*} + |\Delta\theta 1\Delta\theta 2|N_{am}N_{am}{}^{*} + |\theta 1 + \Delta\theta 2|N_{\theta}N_{am}{}^{*}$$

c) sign[$\Delta\theta 1, \Delta\theta 2$] =[+,-] or [-,+]

$$P = N_{\theta}N_{\theta}{}^{*} - |\Delta\theta 1\Delta\theta 2|N_{am}N_{am}{}^{*} \pm |\theta 1 - \Delta\theta 2|N_{\theta}N_{am}{}^{*}$$

The AM suppression is generally defined as 20*log10($\Delta\theta 1$) and 20*log10($\Delta\theta 2$) for each of the two channels when the AM and PM noise are uncorrelated, and for the cross-correlation it is 10*log10($\Delta\theta 1 \cdot \Delta\theta 2$). As an example, when $\Delta\theta 1 = \Delta\theta 2 = 5\,deg$, the AM suppression is 21.2 dB. Both Case 1 and 2 are studied through Transient (SPICE) ADS simulations using $\Delta\theta 1\ and\ \Delta\theta 2\ of\ \pm 5\,deg$ resulting in 21.2 dB AM suppression. The AM level was set 21.2 dB higher than the PM level so that at the outputs of each Phase Detector they would be of equal level of -94 dBV/Hz as an example. It should be noted that $|\Delta\theta|N_{am} = N_{\theta}$ is the Transient simulations.

Transient simulation results

The results of the transient noise simulations of the total cross-correlation power for each of the above cases are as follows:

Case 1: Uncorrelated AM and PM

AM=-94+21.2 dBV/Hz, PM=-94 dBV/Hz at Source. 5 deg phase errors.	Measured cross-correlated power
4-port millimeter-wave measurements	Option 400
Phase error polarity = [+,+] or [-,-]	-91 dBV/Hz. Expected linear: $2N_\theta N_\theta^*$
Phase error polarity = [+,-] or [-,+]	-107 dBV/Hz. Expected linear: 0

The -107 dBV/Hz number above keeps reducing as more averaging is done in the transient simulation. The number reported here is with moving average of 1000 FFT tones.

Case 2: Anti-correlated AM and PM

AM=-94+21.2 dBV/Hz, PM=-94 dBV/Hz at Source. 5 deg phase errors.	Measured cross-correlated power
Phase error polarity = [+,+]	-140 dBV/Hz. Expected linear: 0
Phase error polarity = [-,-]	-88 dBV/Hz. Expected linear: $4N_\theta N_\theta^*$
Phase error polarity = [+,-] or [-,+]	-114 dBV/Hz. Expected linear: $\|\theta 1 - \Delta\theta 2\| N_\theta N_\theta^*$

The -140 dBV/Hz number above doesn't reduce as more averaging is done in the transient simulation as it is already a greatly reduced number due to cancellation of correlated terms. The number reported here is with moving average of 1000 FFT tones.

Transient simulation setup

The schematics and equations used for the transient noise simulations of the AM-PM noise cancellation of the Carrier are shown below. Either common or independent baseband noise sources are used to generate the AM and PM noise at the carrier frequency using DSB modulation.

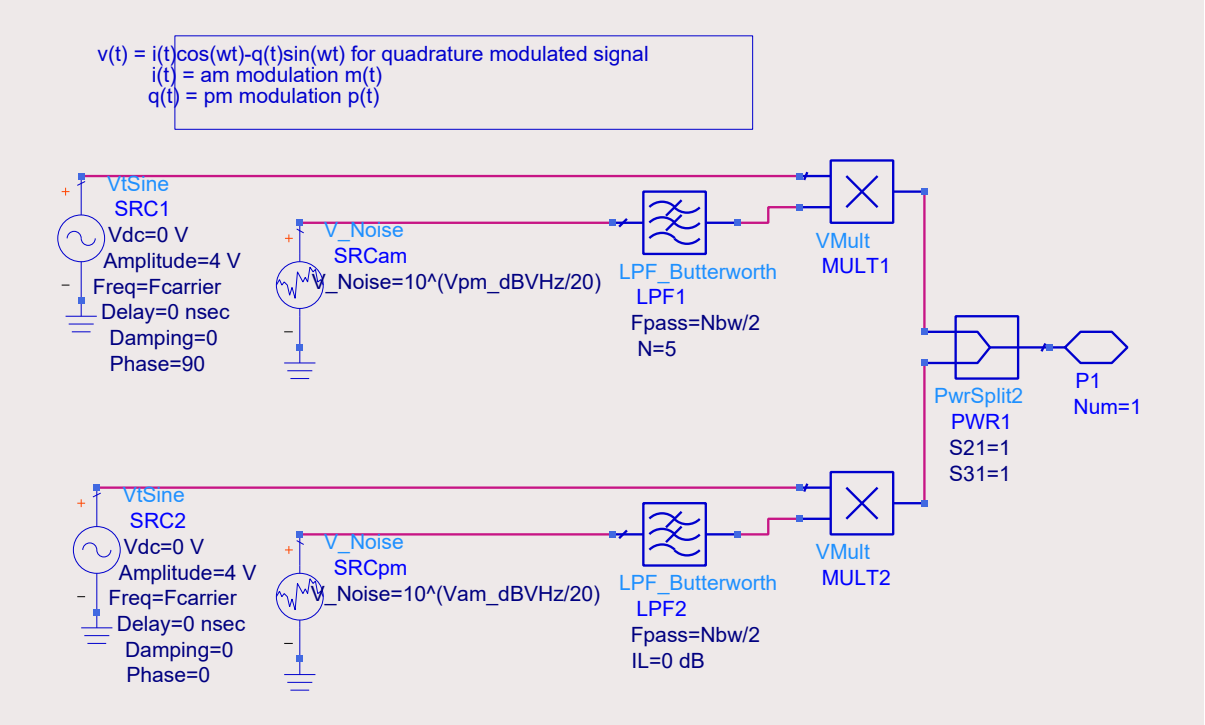


Figure 29. Uncorrelated AM and PM noise

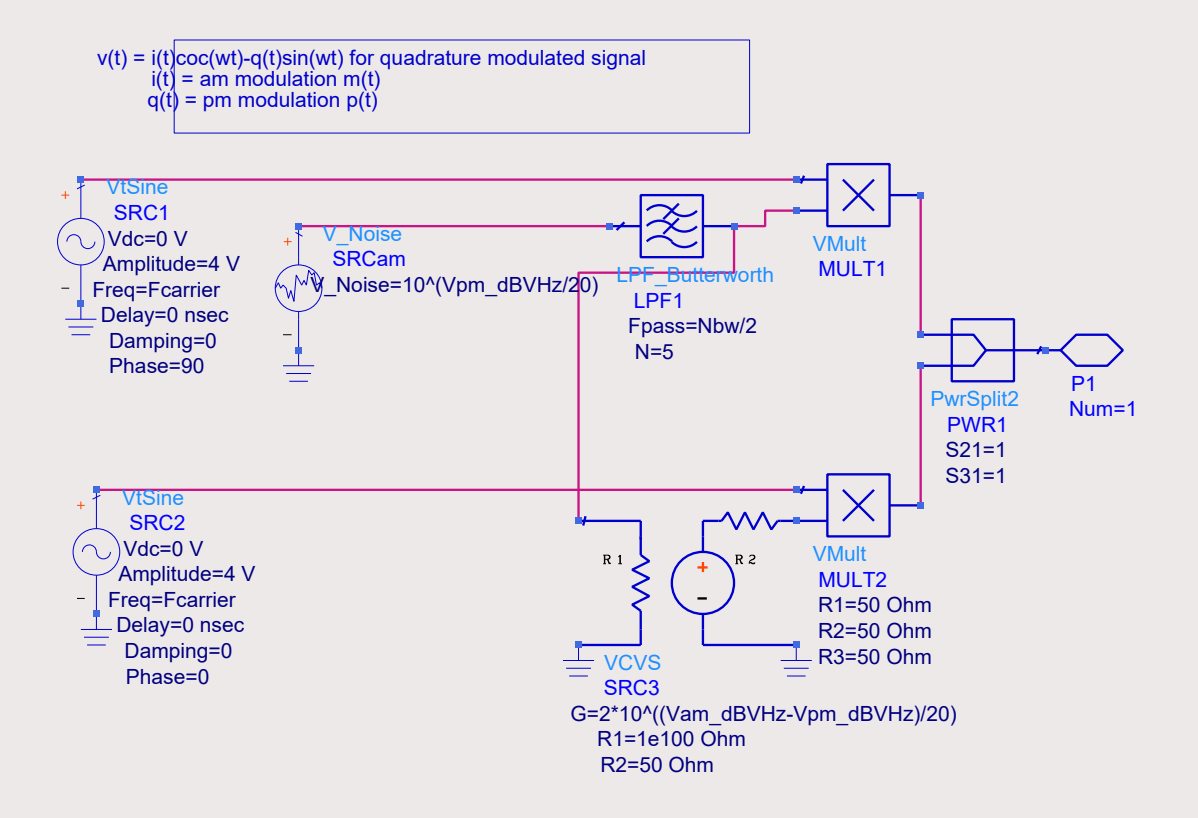


Figure 30. Anti-correlated AM and PM noise. For correlated AM-PM noise, the gain of the VCVS is made negative.

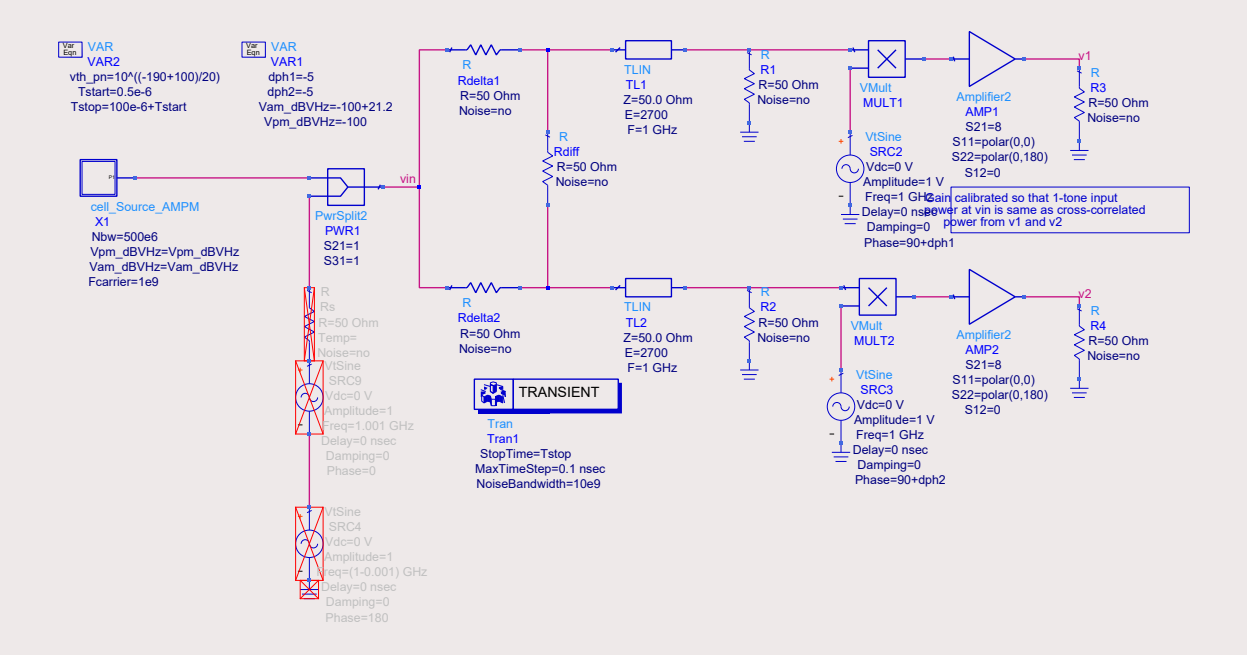


Figure 31. Top-level transient setup for AM and PM noise cancellation

```
Eqn=Tstop[1]-Tstart[1]
Eqntep=1/Tfft
Eqnv_db=10*log10(fstep)

EqnNavg=1000

Eqnmoving_average(fs(v1)*conj(fs(v2)),Navg)
Eqnal_dB=db(sqrt(pTotal))-rbw_db-3

Eqnving_average(fs(vin)*conj(fs(vin)),Navg)
Eqn_dB=db(sqrt(pIn))-rbw_db-3

Eqnving_average(fs(v1)*conj(fs(v1)),Navg)
Eqn_dB=db(sqrt(p1))-rbw_db-3
```

Figure 32. Equations for computing cross-spectral powers from the time-domain noise signal

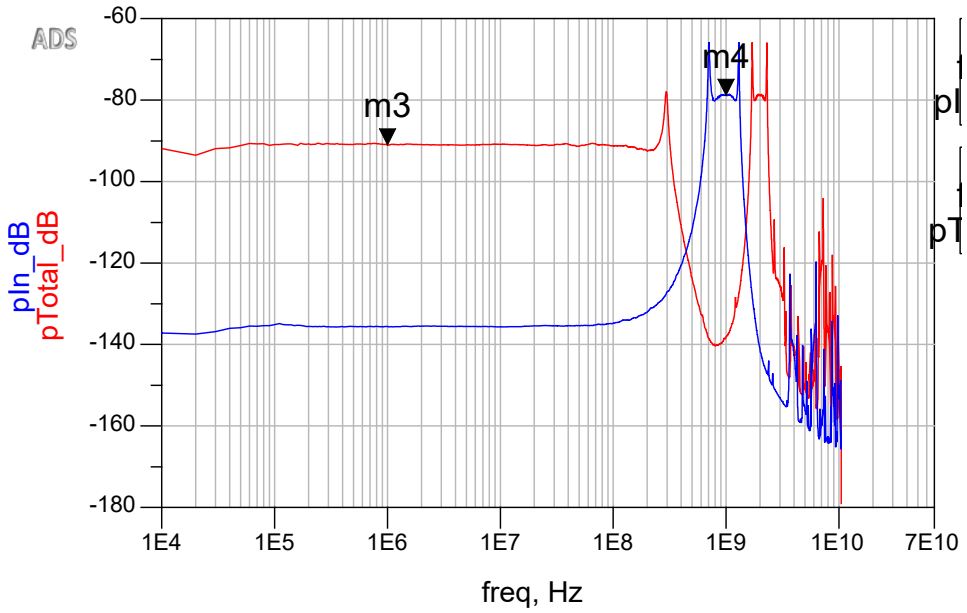


Figure 33. Example of a transient simulation result for Case 1, phase error polarity = [+,+] or [-,-]
Blue: Source PSD of PM and +21.2 dBc AM
Red: Down converted cross-spectrum

Anti-correlated AM with opposite polarity reference phase errors

In this Harmonic Balance simulation, AM noise in the Source is varied relative to the PM noise and the Autocorrelation power of the source is measured as a function of the Quadrature phase error. The results are shown in the figures below. As mentioned earlier, when the Quadrature phase error is of opposite polarities in the two channels, the AM noise auto-correlation power is negative i.e., anti-correlated due to 180 deg phase. However, when combined with PM noise of the Source, the phase is approaches 180 deg only for large AM levels relative to PM noise, or if there is significant Quadrature phase error. For a typical the AM suppression of 20 dB in the Phase Detector the corresponding Quadrature phase error is around 5 deg and is a result of LO and RF self-detection in the down converter. At 5 deg phase error and 21.2 dBc AM level, the Source correlated power has 90 deg phase. In this case the auto-correlated power becomes orthogonal i.e., uncorrelated to the real power value. The phase is always zero if the polarities of quadrature errors are the same for each channel.

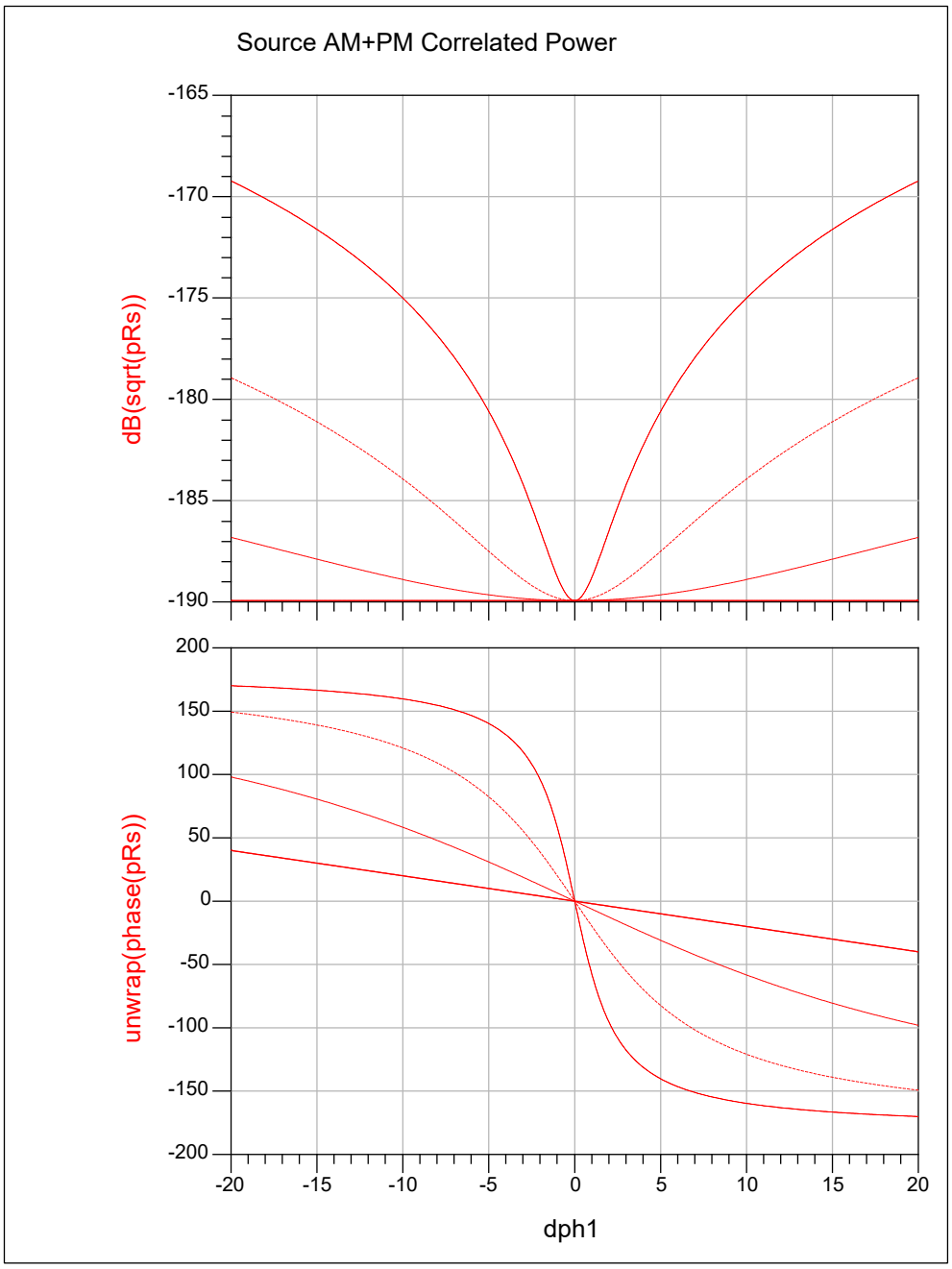


Figure 34. Source correlated power (magnitude and phase) over quadrature phase error with opposite signs in each channel. Traces are for relative AM noise level of 0, 10, 20 and 30 dB above PM noise that is at -190 dBV/Hz. X-axis is quadrature phase error of Channel 1. Channel 2 phase error is negative of Channel 1. At 5 deg phase error and 21.2 dBc AM level, the Source correlated power has 90 deg phase. The phase is always zero if the polarities of quadrature errors are the same for each channel.

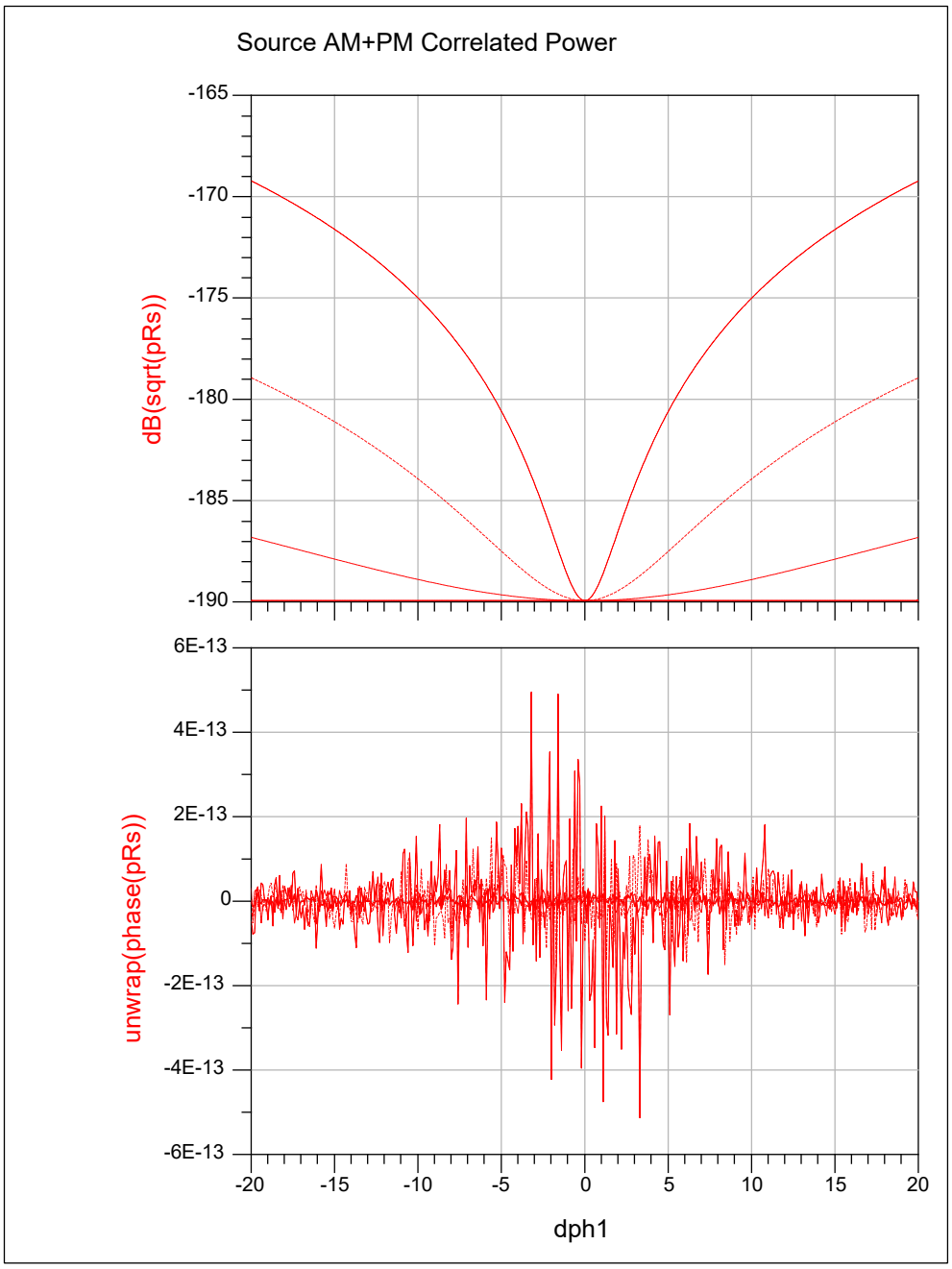


Figure 35. Source correlated power (magnitude and phase) over quadrature phase error with same sign in each channel. Traces are for relative AM noise level of 0, 10, 20 and 30 dB above PM noise that is at -190 dBV/Hz. X-axis is quadrature phase error of Channel 1 and Channel 2. The phase of correlated power is always zero if the polarities of quadrature errors are the same for each channel.

Countermeasures to Cross-Spectral Collapse

Uncorrelated noise sources create correlated or anti-correlated noise terms relative to the DUT correlated noise in the cross-spectral measurements of phase noise and baseband noise. The anti-correlated terms produce cross-spectral collapse.

- The 50 ohms source resistor (Rs) phase noise in the DUT (or Source) is cancelled by the anti-correlated noise from the power divider (WPD or RPD). However, this phase noise of Rs is typically much smaller than the phase noise of the source signal and therefore there is no observable spectral collapse from this except when the source signal phase noise approaches the thermal level.
- The main contributors to Cross-Spectral Collapse are noise leaking from one channel to the other. This is mitigated by increasing the isolation between the channels including the use of isolators and low noise amplifiers after the WPD.
- Resistive Power Divider (RPD) should be avoided for phase noise measurements due to reduced isolation that can show up as cross-spectral nulls from reference noise.
- For baseband noise measurement RPD helps to decrease the RF cable lengths after the power divider to push the null frequencies of the Cross-Spectral Collapse to higher values.
- RPD also extends the lower frequency to 0 Hz that is not possible with WPD for baseband noise measurements.
- For phase noise measurements the RPD increases the spacing between the spectral null frequencies due to the very short electrical lengths inside it for each channel. The absolute position of the nulls are some high odd harmonics of the first null which is typically in the 10 MHz range with reasonable short external cable lengths for N5511A.
- Cross-Spectral Collapse is not observable when the Source correlated noise level or the uncorrelated system noise level (after correlation suppression) is much larger than the total anti-correlated noise level but is clearly visible otherwise. In the latter case the Source noise would be getting close to the system noise floor and/or the uncorrelated noise would have been suppressed significantly to thermal levels or lower.
- Baseband and IF circuit design in dual-channel systems should reduce the actual input port noise and not just the input referred noise. The input port noise leaks to the other channel and leads to cross-spectral collapse.
- “kt” noise of 50 ohms source resistance that is cancelled by the power divider can be added back for baseband noise measurement. System can be calibrated with 50 ohms input termination and corresponding measured “reference” noise can be subtracted from DUT’s measured noise, and then the “kt” noise can be added back.

Conclusion

Theory of Cross-Spectral Collapse is explained theoretically and demonstrated with simulations and measurements, for both baseband noise and phase noise. AM-PM Source noise cancellation is derived and shown in simulations. The main counter measure to cross-spectral collapse is to improve isolation between the two channels of the cross correlation to reduce both correlated and anti-correlated system noise relative to the DUT noise. The channel delays should be minimized to increase spacing between cross spectral nulls, and for baseband noise to push them out to high frequencies.

Appendix

Phase noise definition

A noisy carrier with AM noise $A(t)$ and phase noise $\emptyset(t)$ is shown as:

$$v_n(t) = A(t) \cdot \cos\left[w_c t + \emptyset(t)\right] \quad (1)$$

Expanding Equation 1,

$$v_n(t) = A(t)[\cos(w_c t)\cos\big(\emptyset(t)\big) - \sin(w_c t)\sin\big(\emptyset(t)\big)] \quad (3)$$

Under small angle condition of $|\emptyset(t)| << \frac{\pi}{2}$,

$$v_n(t) = A(t)[\cos(w_c t) - \sin(w_c t)\emptyset(t)] \quad (4)$$

For a narrow measurement bandwidth at offset frequency w_m,

$\emptyset(t) = \emptyset_0 \sin(w_m t)$, $\emptyset_0 << \frac{\pi}{2}$, (5)

$$v_n(t) = A(t)[\cos(w_c t) - \frac{\emptyset_0}{2}\cos(w_c - w_m)t + \frac{\emptyset_0}{2}\cos(w_c + w_m)t] \quad (6)$$

From the above Equation we see there exists two sidebands at frequencies $(w_c - w_m)$ and $(w_c + w_m)$.

The power of each sideband relative to carrier is $\frac{\emptyset_0^2}{4}$ and is defined as the phase noise power $L_v(f_m)$ obtained directly from the signal spectrum under small angle condition. Note that $\omega_m = 2\pi f_m$.

Therefore,

$L_v(f_m) = \frac{\emptyset_0^2}{4}$ defined for small angle condition (7)

The subscript v in $L_v(f_m)$ is a reminder that the phase noise is being derived from the signal spectrum under small angle conditions. Independent of whether the signal satisfies the small angle condition, the power of the phase modulation signal $\emptyset(t) = \emptyset_0 \sin(w_m t)$ is denoted as

$S_\emptyset(f_m) = \frac{\emptyset_0^2}{2}$ (8)

Using Equations (7) and (8) we see that under small angle condition

$L_v(\mathrm{f}) \approx \frac{1}{2} S_\emptyset(f)$ (9)

AM noise definition

The AM noise $A(t)$ can be defined as

$$A(t) = R[1 + m(t)]$$

$$v_n(t) = R[1 + m(t)] \cdot \cos[w_c t + \emptyset(t)]$$

Taking a 1-tone example for $m(t)$

$$m(t) = m_o \cos(w_{am} t)$$

Expanding on the AM noise $A(t)$ the noisy carrier with phase noise $\emptyset(t)$:

$$v_n(t) = R[1 + m_o \cos(w_{am} t)] \cdot \cos[w_c t + \emptyset(t)] \quad (12)$$

$$v_n(t) = R\{\cos[w_c t + \emptyset(t)] + m_o \cos(w_{am} t) \cdot \cos[w_c t + \emptyset(t)]\}$$

$$v_n(t) = R\{\cos[w_c t + \emptyset(t)] + \frac{m_o}{2}\cos[(w_c + w_{am})t + \frac{\emptyset(t)}{2}] + \frac{m_o}{2}\cos[(w_c - w_{am})t + \frac{\emptyset(t)}{2}]\}$$

There are 3 terms: $\cos[w_c t + \emptyset(t)]$ which is the carrier with phase noise, and two AM sideband signals centered at frequencies $(w_c + w_{am})$ and $(w_c - w_{am})$ with amplitude $\frac{m_o}{2}$ and phase modulation $\frac{\emptyset(t)}{2}$. This phase modulation $\frac{\emptyset(t)}{2}$ power is much lower than that of the AM sideband carrier and therefore can be ignored relative to the sideband carriers. The resultant signal is

$$v_n(t) = R\{\cos[w_c t + \emptyset(t)] + \frac{m_o}{2}\cos[(w_c + w_{am})t] + \frac{m_o}{2}\cos[(w_c - w_{am})t]\} \quad (13)$$

The figure below depicts the AM and Phase Noise for 1-tone AM and Phase modulations at arbitrary frequencies w_{am} and w_m respectively. The phase noise term $\cos[w_c t + \emptyset(t)]$ is depicted by Equation 6.

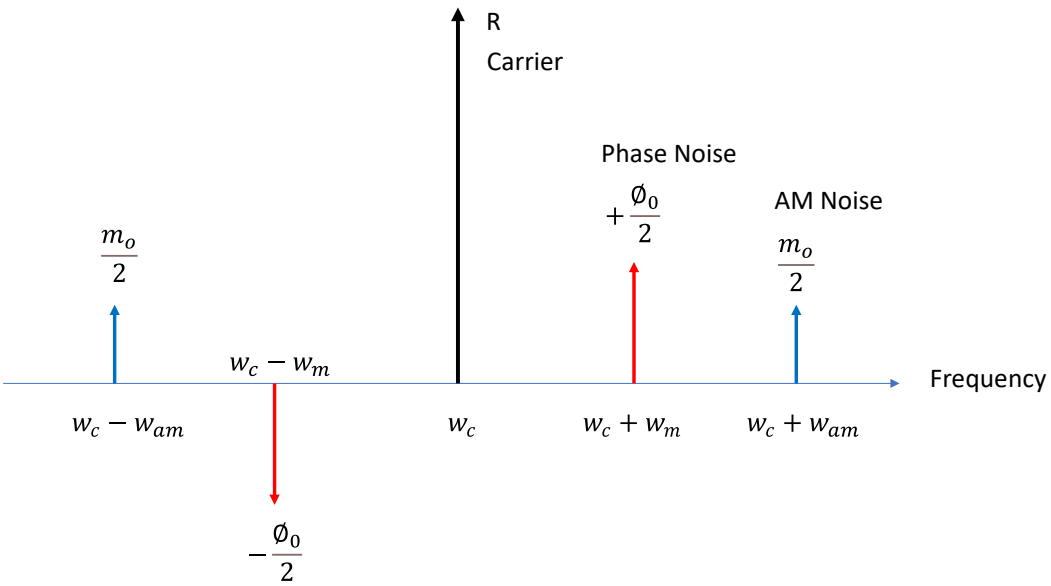


Figure 1. AM and PM spurs with modulation frequencies w_{am} and w_m respectively

Thermal noise

The rms voltage amplitude of each of the four tones in the above figure when considered for thermal noise at 300K is -190 dBV/Hz when their individual powers are considered in 1Hz bandwidth each. The total rms voltage in 1Hz bandwidth is -187 dBV/Hz and is the rms value of the AM and PM noise voltages. If the carrier voltage is 0 dBV then the phase noise $L_v(\mathrm{f})$ is -190 dBc/Hz from the thermal noise based on the ratio of the sideband power to the carrier power. Of course, the PSD of phase noise $S_\emptyset(f_m) = \frac{\emptyset_0^2}{2}$ is due to the contribution of both the upper and lower sidebands and the rms value of their combined voltage is -187 dBV/Hz.

Power spectral density

The distribution of average power of a signal in the frequency domain is called the Power Spectral Density (PSD) or Power Density (PD) or power density spectrum. The power spectral density is denoted by S(ω) and is given by

$$S(\omega) = \lim_{\tau \to \infty} \frac{|X(\omega)|^2}{\tau}$$

Autocorrelation

The autocorrelation function gives the measure of similarity between a signal and its time-delayed version. The autocorrelation function of power (or periodic) signal x(t) with any time period *T* is given by,

$$R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-(T/2)}^{T/2} x(t)\ x^*(t-\tau)\ dt$$

Where, τ is called the *delay parameter.*

Relation between PSD and autocorrelation function

The power spectral density function S(ω) and the autocorrelation function R(τ) of a power signal form a Fourier transform pair, i.e.,

$$R(\tau) \overset{FT}{\leftrightarrow} S(\omega)$$

Wiener-Khintchine Theorem

C. A. Bouman: Digital Image Processing - January 7, 2020

For a well behaved stationary random process the power spectrum is equal to the Fourier transform of the autocorrelation function.

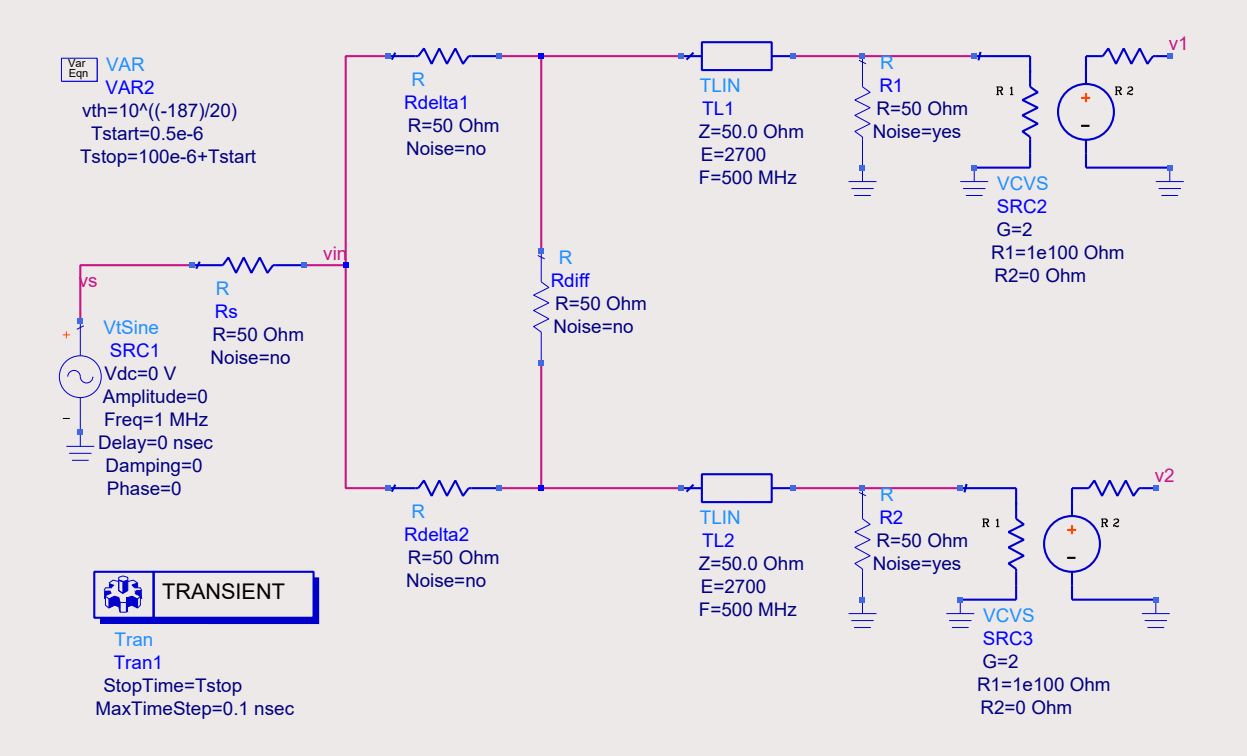
$$S_x(e^{j\omega}) = \sum_{k=-\infty}^{\infty} R_x(k)e^{-j\omega k}$$

Sloppy proof:

$$\begin{aligned}
S_x(e^{j\omega}) &= \lim_{N\to\infty} \frac{1}{2N+1} E[|X_N(e^{j\omega})|^2] \\
&= \lim_{N\to\infty} \frac{1}{2N+1} E\left[\left(\sum_{n=-N}^{N} x(n)e^{-j\omega n}\right)\left(\sum_{k=-N}^{N} x(k)e^{-j\omega k}\right)^*\right] \\
&= \lim_{N\to\infty} \frac{1}{2N+1} E\left[\sum_{n=-N}^{N}\sum_{k=-N}^{N} x(n)x(k)e^{-j\omega(n-k)}\right] \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N}\sum_{k=-N}^{N} E[x(n)x(k)]e^{-j\omega(n-k)} \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N}\sum_{k=-N}^{N} R_x(n-k)e^{-j\omega(n-k)} \\
&= \lim_{N\to\infty} \lim_{M\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N}\sum_{k=-M}^{M} R_x(n-k)e^{-j\omega(n-k)} \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \lim_{M\to\infty} \sum_{k=-M}^{M} R_x(n-k)e^{-j\omega(n-k)} \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\sum_{k=-\infty}^{\infty} R_x(k)e^{-j\omega k}\right) \\
&= \left(\sum_{k=-\infty}^{\infty} R_x(k)e^{-j\omega k}\right) \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1 \\
&= \sum_{k=-\infty}^{\infty} R_x(k)e^{-j\omega k}
\end{aligned}$$

Transient baseband simulations

Schematic



Eqn Tstop[1]-Tstart[1]
Eqn fstep=1/Tfft
Eqn fstep_db=10*log10(fstep)

Eqn Navg=1000

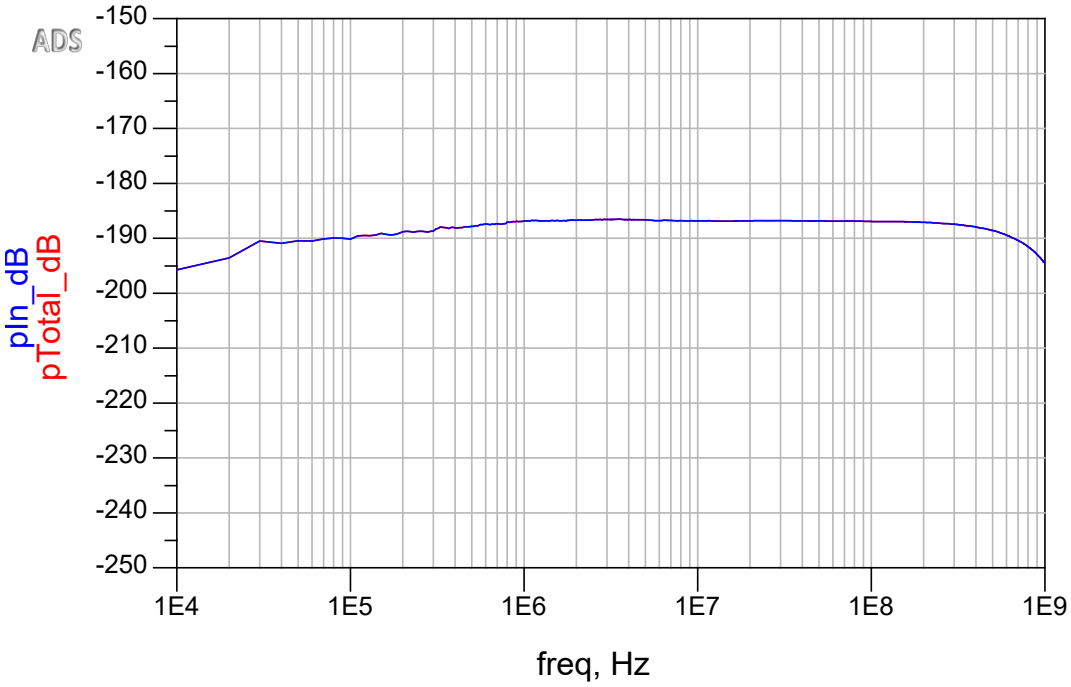
Eqn pTotal=moving_average(fs(v1)*conj(fs(v2)),Navg)
Eqn rbw_db=db(sqrt(pTotal))-rbw_db-3

Eqn pIn=moving_average(fs(vin)*conj(fs(vin)),Navg)
Eqn pIn_dB=db(sqrt(pIn))-rbw_db-3

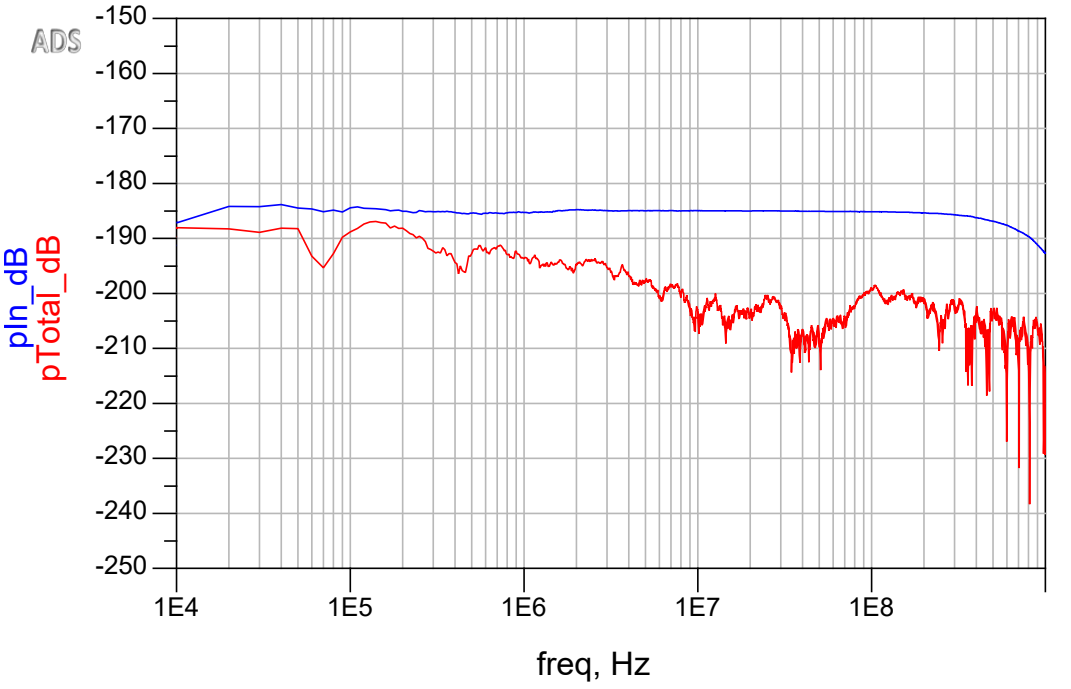
Eqn p1=moving_average(fs(v1)*conj(fs(v1)),Navg)
Eqn p1_dB=db(sqrt(p1))-rbw_db-3

Above is simulation setup for only R1 and R2 noise

Only Rs

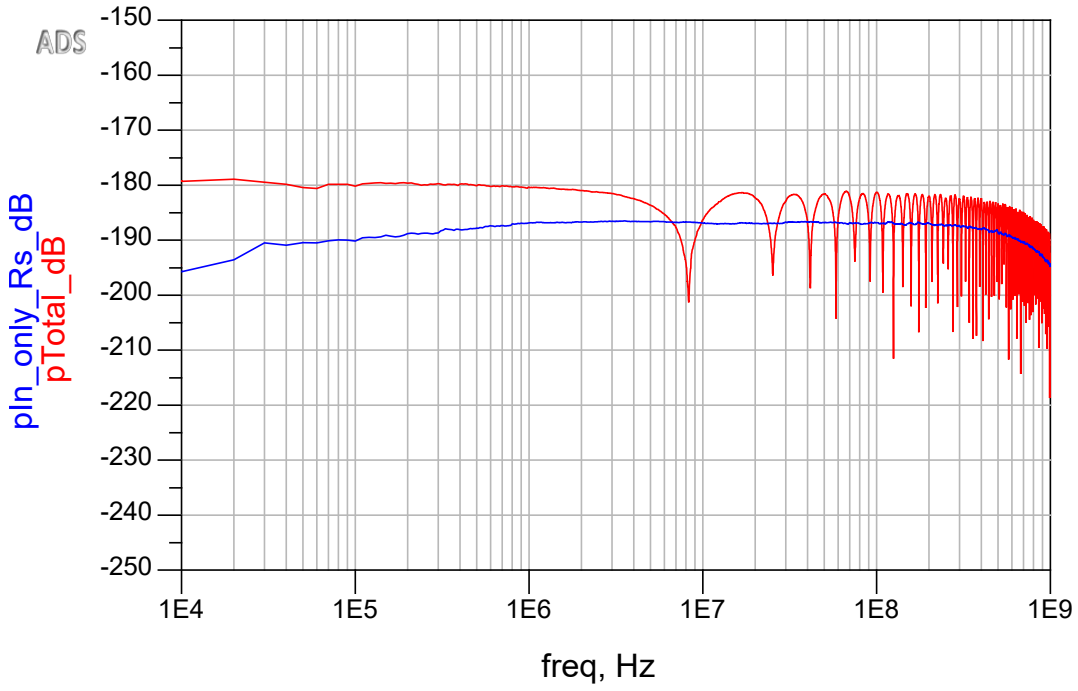


Only Rs and RPD

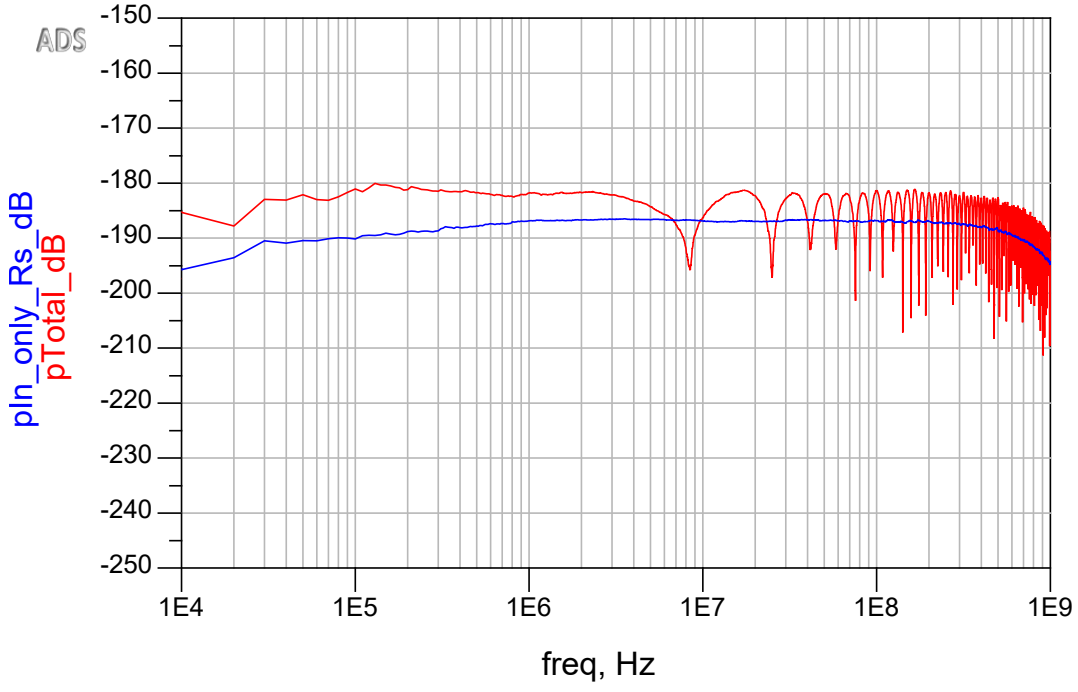


Moving Avg = 10k tones

Only R1 and R2 noise

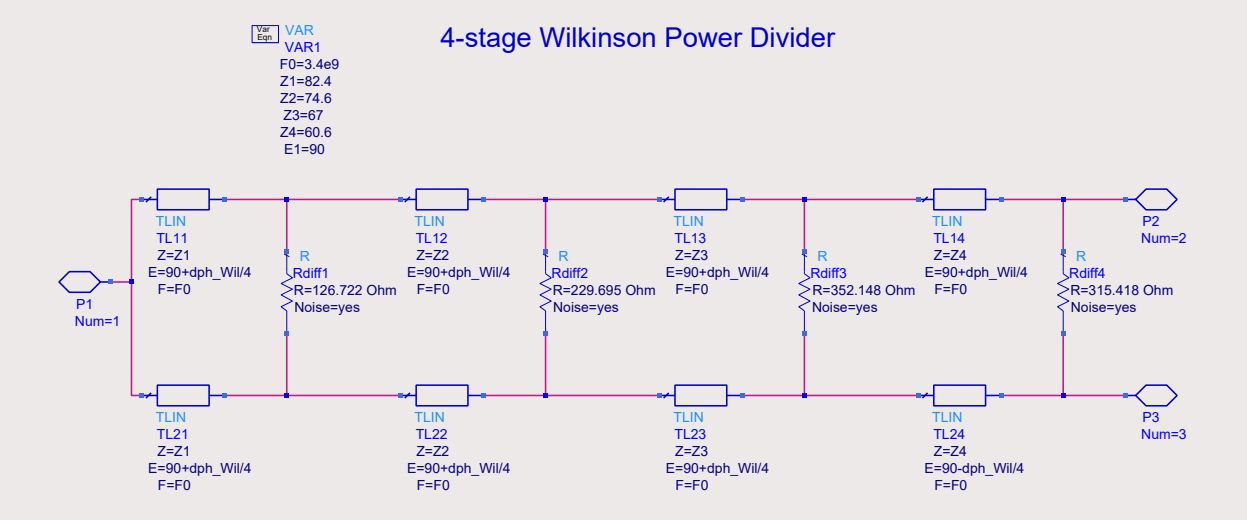


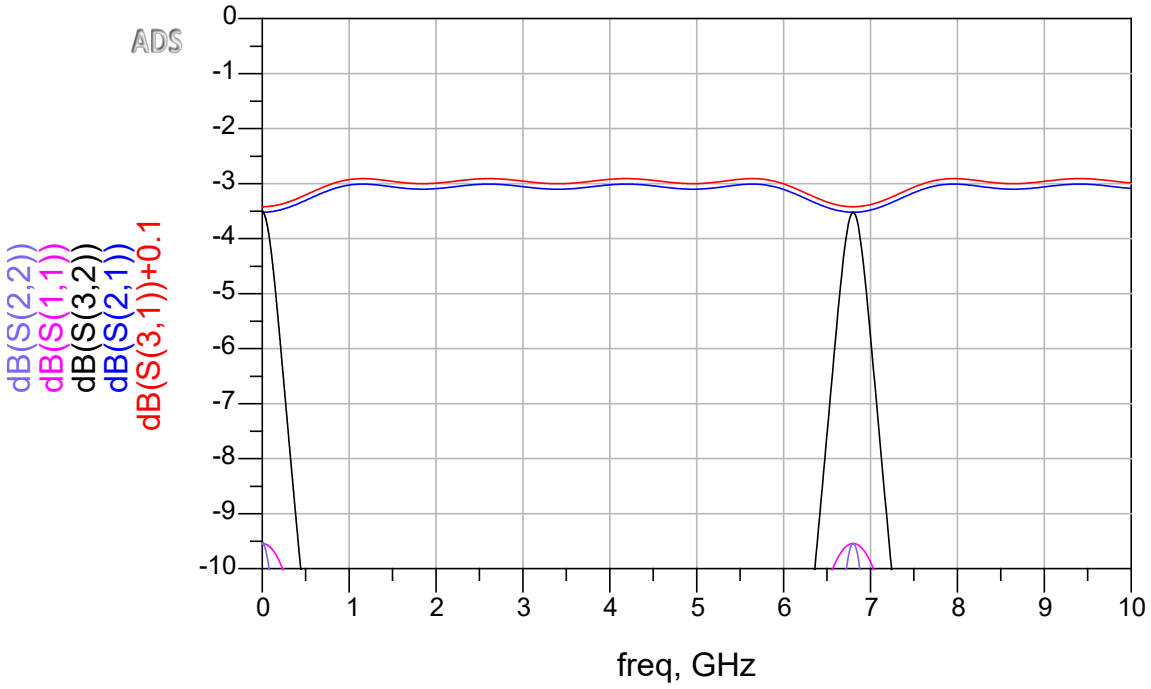
All Resistors

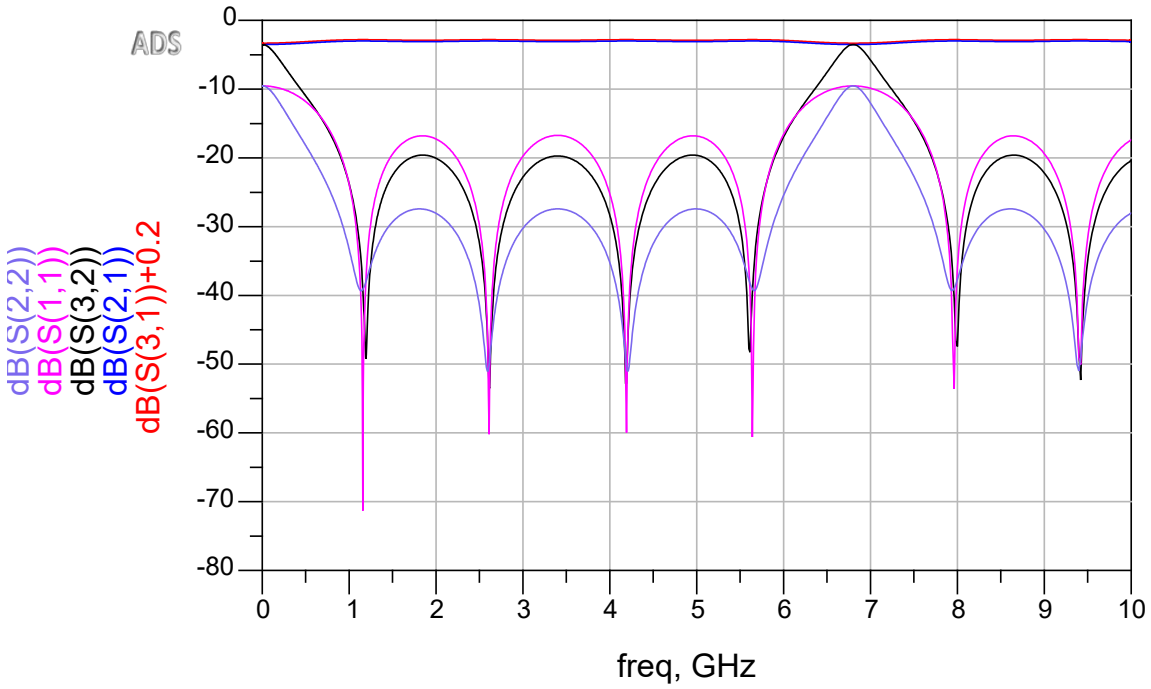


4-Stage Wilkinson Power Divider

The design and simulated responses of the 4-Stage Wilkinson Power Divider are shown below. It has 3 dB loss and 20 dB isolation at 5 GHz.







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