A Refresher Course on Windowing and Measurements

In these days of digital instrumentation and PCs, it is easy to forget that physical phenomena are analog and that windows are not always operating systems. Windowing and digitization meet in the process of dynamic signal analysis. This article explains the need for windowing and describes the advantages of common windows used by FFT-based analyzers.

What happens if you don't do windows

FFT analyzers transform data from the time domain to the frequency domain by computing the fast Fourier transform. It is based on the Fourier integral ($\int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}$), but is a form that can be computed numerically. The integral requires that a continuous signal be integrated over infinite time. Out here in the real world, of course, we want our results in finite time. And because computers work with numbers, we need to digitize the waveform, which makes it discrete in time. Both of these changes to the signal result in errors in the computed frequency spectrum. Sampling the signal at discrete times can cause *aliasing* (which can show up as "phantom" signals on the display). Changing the limits of integration from infinity to a finite length can cause an error known as *leakage* (which appears as energy from a particular point in the spectrum being "smeared" up and down across the spectrum).

Since it's impossible to measure a signal for an infinite time, the analyzer changes the limits of integration to the length of time it takes to collect a block of samples. This block of samples is called a *time record*. The FFT requires that the signal within the time record be repeated over and over again throughout time. If the repeated time records actually look like the original signal, no leakage will occur. If, on the other hand, they do not look the same, leakage will occur. Applying a window function to the data can help decrease the effects of leakage in the frequency domain.

If the time record contains an integral number of cycles of a continuous waveform, such as a sine wave, the waveform is said to be periodic in the time record. The value of the signal at the beginning of the time record is equal to the value at the end of the time record. When the time records are placed end to end, the start and finish points align exactly. The time record is indeed repeated over and over, and the Fourier integral can be computed accurately. No leakage occurs.
Figure 1 shows a signal that is periodic in the time record.

Now, if a continuous waveform is not periodic in the time record, leakage will occur. When time records are placed end to end, the start and finish points don't align. This is like adding an instantaneous step to the waveform. Because an instantaneous step contains an infinite number of frequencies, these added frequencies contaminate the computed result. The power from the instantaneous step is spread across the spectrum, resulting in the leakage shown in figure 2.

FFT analyzers use windowing to reduce the effect of leakage, thereby improving the results in the frequency domain. The various window functions have different advantages, and it's important to choose the correct one for each measurement. The most common are the Hann, flattop, uniform, and force/response windows.

The Hann window

Use the Hann window to identify closely spaced frequency components or to measure broadband noise.
The Hann function (it's named after a fellow named Hann, in case you're curious) looks much like a bell-shaped curve, varying from zero on the ends to unity in the center. Figure 3 illustrates the effect of the Hann window. Compare these traces to the same signal shown unwindowed in figure 2.

The Hann window's good frequency resolution comes at the cost of poor amplitude accuracy, however. Because the FFT acts like a set of parallel filters, signals not at the center of the bandpass filters see some attenuation. When you use the Hann window, a frequency falling midway between the filter's center frequency is attenuated by up to 1.5 dB.

The flattop window

If accurate measurement of amplitude is important, use the flattop window (figure 4). The flattop provides excellent amplitude accuracy, with amplitude variations reduced to less than 0.1 dB. But, again, you'll face a tradeoff. The flattop window gives you excellent amplitude accuracy but gives up some frequency resolution. The skirts of the filters are so wide that you lose some ability to resolve small frequency components spaced near large components.

The uniform window

The uniform window is really no window at all. It is sometimes called the "boxcar function" because it looks like a boxcar, a pulse that is unity for all values of time. The uniform window provides the best frequency resolution and amplitude accuracy, but can only be used if the measured signal is periodic in the time record. This condition is rarely met with naturally occurring signals, but can be met in controlled testing. Because the frequency of any applied signal can be known, it is possible to make sure that the signal is periodic in the time record and the frequency is centered in one of the FFT's bandpass filters. Because the signal is periodic in the time record, there is no leakage. And because it is centered in the bandpass filter, no amplitude inaccuracy is caused by attenuation. (The measurements in figures 1 and 2 were made with the uniform window.)

The force and response windows

Force and response windows are used for a specific application—computing the transfer function of a mechanical structure using an impulsive force excitation (see figure 5). A hammer or other impact device equipped with a force transducer is used to strike the structure and an accelerometer measures the structure's response vibration. The force transducer is connected to one input channel of the analyzer and the response accelerometer is connected to the second channel (in a simple single-input, single-output measurement).

The time record for the force should include only the impact with the structure. The movement of the hammer before and after striking the structure can cause stray signals in the time record. The force window (applied to the signal from the hammer) is unity where the impact signal is valid and zero everywhere else to eliminate noise.

The response window is applied to the accelerometer signal from the structure. This window ensures that the response signal decays to zero by the end of the time record. In fact, it's sometimes called the exponential window.

Do windows—and do them right

Even in these quick examples, you can see the effect windows have on your measurements, and how important it is to select windows that match your application and your test goals. So take a second or two before each measurement and think about what you're trying to accomplish. Selecting the right window will help you get the best answers possible. ■

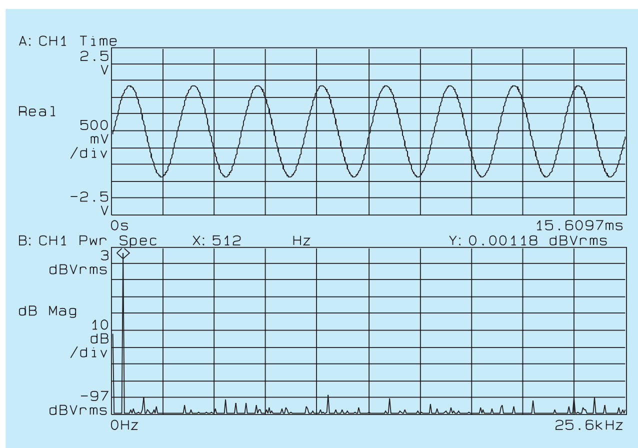

Figure 1: Signal periodic in the time record

The sine wave in the top trace is periodic in the time record, so no window (the uniform window, actually) is needed. The lower trace shows the clean frequency domain representation of this signal, with no noticeable leakage. Compare this with the signal in figure 2.

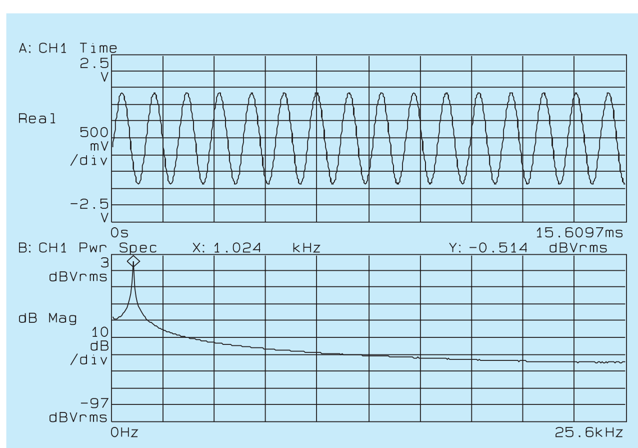

Figure 2: Signal not periodic in the time record

In this case, the sine wave is not periodic, and the bottom trace shows what happens if no window is used in the measurement. The energy spread across the spectrum is called leakage.

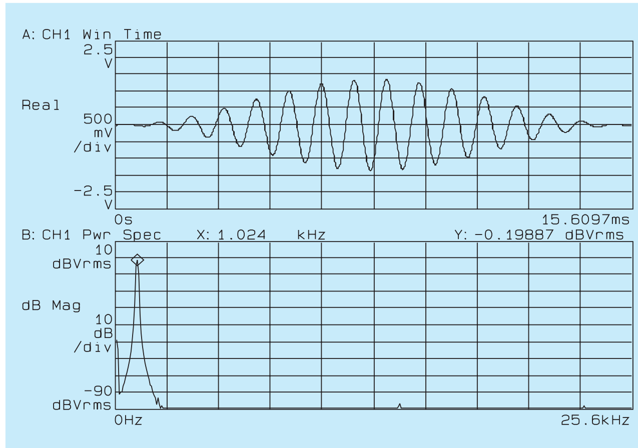

Figure 3: Applying the Hann window

Here's the effect the Hann window has on the nonperiodic signal from figure 2. The window reduced the leakage considerably and offers good frequency resolution, at the expense of some amplitude accuracy.

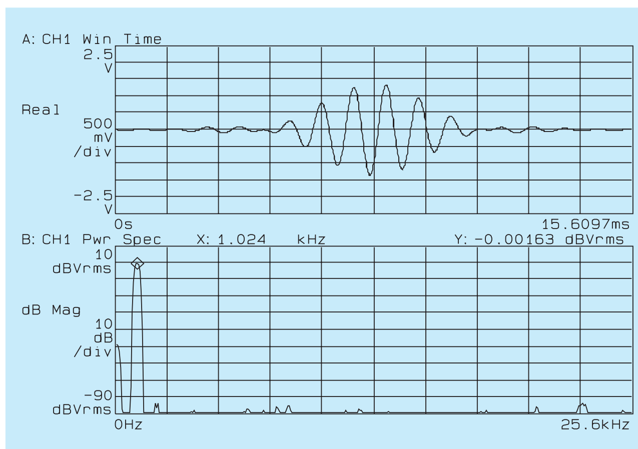

Figure 4: Applying the flattop window

The flattop offers better amplitude accuracy, but you can see it isn't as good as the Hann window at resolving closely spaced frequencies.

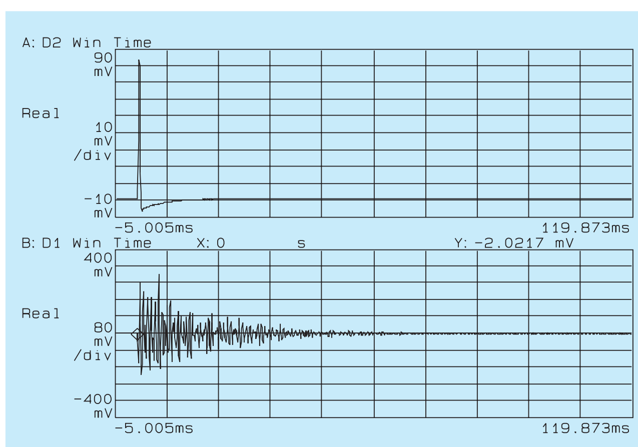

Figure 5: Applying the force and response windows

The top trace shows the effect of the force window; everything after the impact is set to zero. The bottom trace shows the effect of the response window, which forces the signal to decay exponentially until it reaches zero.